Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.3-e-x-^m-a+b-sinh-c+d-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [102]. This is test number [161].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (102)	% 0. (0)
Mathematica	% 99.02 (101)	% 0.98 (1)
Maple	% 78.43 (80)	% 21.57 (22)
Maxima	% 81.37 (83)	% 18.63 (19)
Fricas	% 76.47 (78)	% 23.53 (24)
Sympy	% 28.43 (29)	% 71.57 (73)
Giac	% 42.16 (43)	% 57.84 (59)

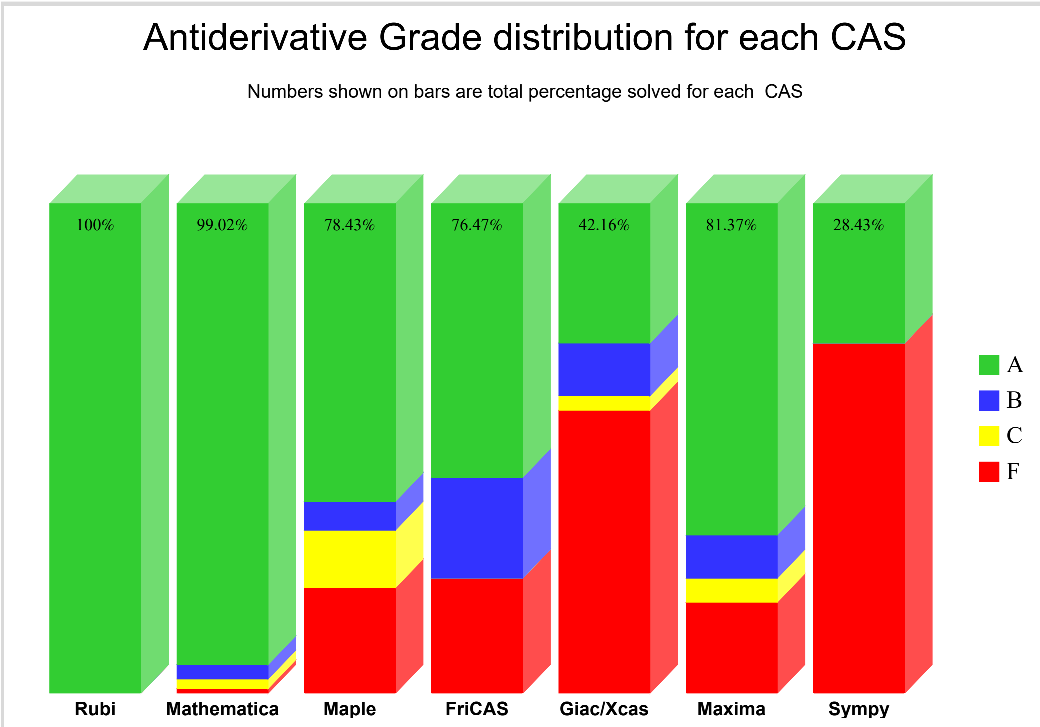
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

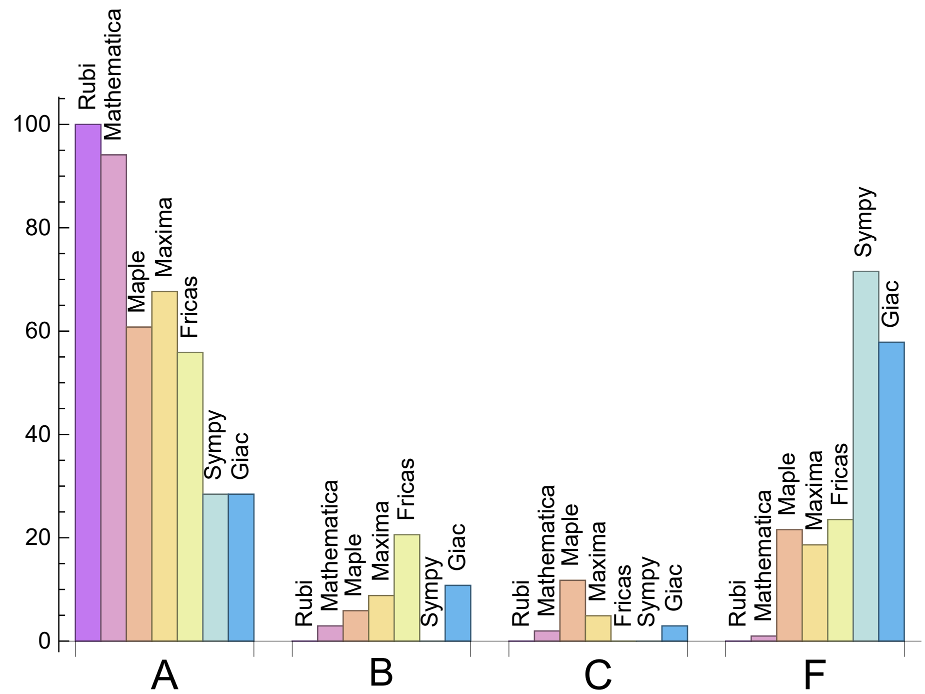
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	94.12	2.94	1.96	0.98
Maple	60.78	5.88	11.76	21.57
Maxima	67.65	8.82	4.9	18.63
Fricas	55.88	20.59	0.	23.53
Sympy	28.43	0.	0.	71.57
Giac	28.43	10.78	2.94	57.84

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	85.72	0.89	68.	1.
Mathematica	1.55	89.1	0.91	65.	0.93
Maple	0.07	103.38	1.08	66.5	1.08
Maxima	1.09	131.78	1.72	80.	1.21
Fricas	1.59	364.51	4.12	184.	3.47
Sympy	4.13	45.03	0.84	29.	1.
Giac	1.03	231.37	1.85	68.	1.64

1.4 list of integrals that has no closed form antiderivative

{23, 27, 40, 56, 74, 75, 77, 79, 83, 91, 92}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {78}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

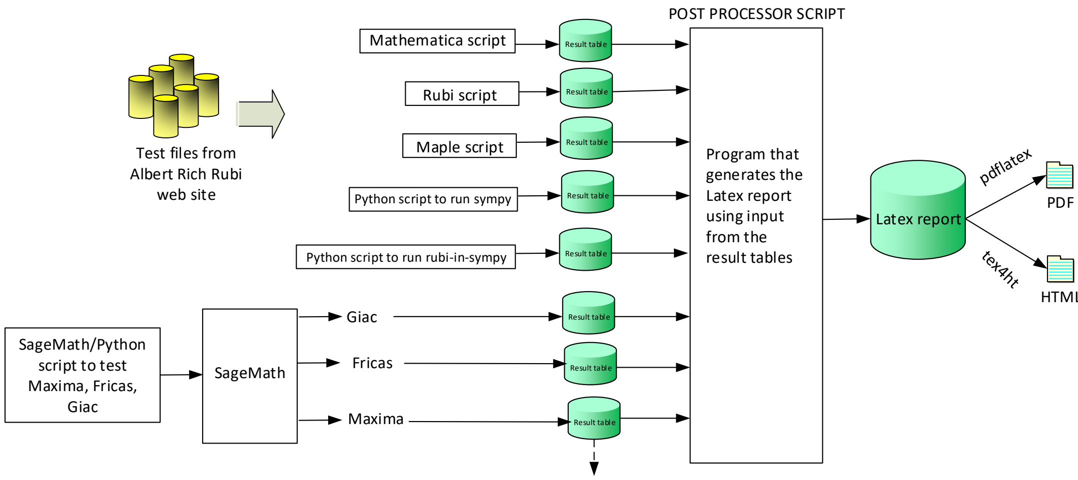
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

B grade: { 3, 24, 53 }

C grade: { 101, 102 }

F grade: { 37}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 61, 67, 72, 74, 75, 77, 79, 83, 84, 85, 86, 87, 91, 92, 95, 100 }

B grade: { 35, 36, 93, 94, 98, 99 }

C grade: { 26, 39, 55, 58, 59, 60, 62, 63, 82, 88, 89, 90 }

F grade: { 24, 25, 37, 38, 53, 54, 64, 65, 66, 68, 69, 70, 71, 73, 76, 78, 80, 81, 96, 97, 101, 102 }

2.1.4 Maxima

A grade: { 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 83, 84, 86, 87, 91, 92, 93, 94, 98, 99, 100

B grade: { 1, 2, 4, 17, 22, 88, 89, 90, 95 }

C grade: { 34, 35, 36, 50, 52 }

F grade: { 24, 25, 26, 37, 38, 39, 53, 54, 55, 76, 78, 80, 81, 82, 85, 96, 97, 101, 102 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 42, 44, 46, 47, 48, 50, 52, 56, 57, 67, 72, 74, 75, 77, 79, 83, 87, 88, 90, 91, 92, 93, 94, 95, 98, 99, 100 }

B grade: { 2, 6, 9, 13, 16, 20, 22, 41, 43, 45, 49, 51, 61, 84, 85, 86, 89, 96, 97, 101, 102 }

C grade: { }

F grade: { 37, 38, 39, 53, 54, 55, 58, 59, 60, 62, 63, 64, 65, 66, 68, 69, 70, 71, 73, 76, 78, 80, 81, 82 }

2.1.6 Sympy

A grade: { 1, 3, 8, 10, 15, 17, 22, 23, 27, 28, 32, 33, 34, 35, 36, 40, 48, 50, 52, 56, 57, 74, 83, 91, 92, 93, 94, 95, 100 }

B grade: { }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 51, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 96, 97, 98, 99, 101, 102 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 9, 11, 12, 16, 17, 18, 19, 22, 23, 27, 28, 33, 40, 48, 56, 57, 74, 75, 77, 79, 83, 91, 92, 100 }

B grade: { 7, 8, 10, 14, 15, 21, 93, 94, 95, 98, 99 }

C grade: { 88, 89, 90 }

F grade: { 6, 13, 20, 24, 25, 26, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 87, 96, 97, 101, 102 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	45	109	69	36	63
normalized size	1	1.	0.91	1.32	3.21	2.03	1.06	1.85
time (sec)	N/A	0.036	0.034	0.013	1.035	1.985	1.127	1.391

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	74	149	539	0	101
normalized size	1	1.	0.97	1.07	2.16	7.81	0.	1.46
time (sec)	N/A	0.041	0.068	0.048	0.993	2.114	0.	1.307

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	14	18	31	19	34
normalized size	1	1.	2.07	0.93	1.2	2.07	1.27	2.27
time (sec)	N/A	0.016	0.01	0.002	1.136	1.996	0.259	1.252

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	116	159	0	55
normalized size	1	1.	0.85	0.75	2.19	3.	0.	1.04
time (sec)	N/A	0.018	0.038	0.017	1.143	1.912	0.	1.205

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	104	0	32
normalized size	1	1.	0.92	1.08	1.28	4.16	0.	1.28
time (sec)	N/A	0.035	0.014	0.012	1.252	1.586	0.	1.117

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	70	73	529	0	0
normalized size	1	1.	1.06	1.06	1.11	8.02	0.	0.
time (sec)	N/A	0.035	0.064	0.023	1.117	1.811	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	58	53	166	0	147
normalized size	1	1.	0.9	1.38	1.26	3.95	0.	3.5
time (sec)	N/A	0.092	0.04	0.017	1.274	1.685	0.	1.155

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	42	55	80	142	78	158
normalized size	1	1.	0.82	1.08	1.57	2.78	1.53	3.1
time (sec)	N/A	0.05	0.102	0.029	1.09	1.671	2.555	1.176

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	101	90	128	1126	0	131
normalized size	1	1.	1.02	0.91	1.29	11.37	0.	1.32
time (sec)	N/A	0.096	0.224	0.056	1.727	1.762	0.	1.17

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	51	68	60	76
normalized size	1	1.	0.87	1.1	1.65	2.19	1.94	2.45
time (sec)	N/A	0.027	0.024	0.006	1.134	1.738	0.636	1.147

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	51	76	225	0	78
normalized size	1	1.	1.1	0.65	0.97	2.88	0.	1.
time (sec)	N/A	0.046	0.076	0.029	1.649	1.795	0.	1.168

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	34	42	138	0	47
normalized size	1	1.	0.89	0.92	1.14	3.73	0.	1.27
time (sec)	N/A	0.059	0.02	0.033	1.352	1.86	0.	1.185

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	94	86	82	1058	0	0
normalized size	1	1.	1.07	0.98	0.93	12.02	0.	0.
time (sec)	N/A	0.068	0.226	0.038	1.17	1.862	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	69	49	216	0	170
normalized size	1	1.	0.81	1.21	0.86	3.79	0.	2.98
time (sec)	N/A	0.121	0.091	0.034	1.387	1.759	0.	1.138

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	93	135	234	92	247
normalized size	1	1.	0.73	1.18	1.71	2.96	1.16	3.13
time (sec)	N/A	0.083	0.128	0.041	1.094	1.779	4.684	1.192

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	184	157	219	2429	0	224
normalized size	1	1.	1.15	0.98	1.37	15.18	0.	1.4
time (sec)	N/A	0.138	0.305	0.07	1.828	1.895	0.	1.239

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	84	116	44	76
normalized size	1	1.	1.	0.85	2.55	3.52	1.33	2.3
time (sec)	N/A	0.033	0.014	0.006	1.055	1.783	1.409	1.226

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	136	86	123	369	0	128
normalized size	1	1.	1.09	0.69	0.98	2.95	0.	1.02
time (sec)	N/A	0.072	0.137	0.036	1.641	1.8	0.	1.264

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	68	231	0	68
normalized size	1	1.	0.89	1.	1.24	4.2	0.	1.24
time (sec)	N/A	0.094	0.034	0.038	1.322	1.643	0.	1.279

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	204	149	138	2395	0	0
normalized size	1	1.	1.5	1.1	1.01	17.61	0.	0.
time (sec)	N/A	0.11	0.329	0.052	1.372	1.869	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	120	78	385	0	301
normalized size	1	1.	0.99	1.32	0.86	4.23	0.	3.31
time (sec)	N/A	0.214	0.117	0.041	1.208	1.74	0.	1.284

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	52	170	398	94	146
normalized size	1	1.	1.	0.78	2.54	5.94	1.4	2.18
time (sec)	N/A	0.047	0.025	0.045	1.03	1.688	13.321	1.311

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	2.379	0.039	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	735	0	0	745	0	0
normalized size	1	1.	3.43	0.	0.	3.48	0.	0.
time (sec)	N/A	0.201	12.765	0.135	0.	1.929	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	512	0	0
normalized size	1	1.	1.13	0.	0.	3.79	0.	0.
time (sec)	N/A	0.148	0.615	0.092	0.	1.833	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	98	77	0	355	0	0
normalized size	1	1.	1.03	0.81	0.	3.74	0.	0.
time (sec)	N/A	0.068	0.153	0.069	0.	1.779	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	2.711	0.041	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	31	19	34
normalized size	1	1.	1.	0.93	1.2	2.07	1.27	2.27
time (sec)	N/A	0.02	0.008	0.002	1.033	1.734	1.143	1.233

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	130	63	212	0	0
normalized size	1	1.	0.9	1.67	0.81	2.72	0.	0.
time (sec)	N/A	0.143	0.065	0.036	1.166	1.72	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	93	59	190	0	0
normalized size	1	1.	0.9	1.55	0.98	3.17	0.	0.
time (sec)	N/A	0.107	0.044	0.03	1.159	1.722	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	56	49	135	0	0
normalized size	1	1.	1.	1.7	1.48	4.09	0.	0.
time (sec)	N/A	0.076	0.02	0.029	1.158	1.7	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	32	95	17	0
normalized size	1	1.	1.	1.29	1.52	4.52	0.81	0.
time (sec)	N/A	0.032	0.015	0.022	1.214	1.71	1.672	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	30	15	34
normalized size	1	1.	1.	1.08	1.38	2.31	1.15	2.62
time (sec)	N/A	0.017	0.005	0.004	1.052	1.624	1.564	1.257

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	44	65	73	29	0
normalized size	1	1.	1.	1.52	2.24	2.52	1.	0.
time (sec)	N/A	0.03	0.026	0.009	1.184	1.635	3.088	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	94	63	96	46	0
normalized size	1	1.	0.85	2.04	1.37	2.09	1.	0.
time (sec)	N/A	0.054	0.044	0.007	1.251	1.715	5.233	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	48	165	65	116	61	0
normalized size	1	1.	0.77	2.66	1.05	1.87	0.98	0.
time (sec)	N/A	0.08	0.058	0.009	1.238	1.785	9.017	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	180.001	0.075	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.252	0.058	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	70	0	0	0	0
normalized size	1	1.	0.94	1.04	0.	0.	0.	0.
time (sec)	N/A	0.088	0.075	0.034	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.085	0.032	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	138	84	795	0	0
normalized size	1	1.	0.98	1.33	0.81	7.64	0.	0.
time (sec)	N/A	0.09	0.11	0.059	1.248	1.805	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	93	59	215	0	0
normalized size	1	1.	0.9	1.5	0.95	3.47	0.	0.
time (sec)	N/A	0.113	0.042	0.031	1.241	1.773	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	103	78	697	0	0
normalized size	1	1.	0.98	1.2	0.91	8.1	0.	0.
time (sec)	N/A	0.066	0.088	0.036	1.18	1.781	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	39	58	53	158	0	0
normalized size	1	1.	0.93	1.38	1.26	3.76	0.	0.
time (sec)	N/A	0.082	0.025	0.023	1.219	1.713	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	70	70	96	606	0	0
normalized size	1	1.	1.04	1.04	1.43	9.04	0.	0.
time (sec)	N/A	0.044	0.067	0.031	1.221	1.787	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	32	105	0	0
normalized size	1	1.	1.	1.08	1.28	4.2	0.	0.
time (sec)	N/A	0.033	0.014	0.023	1.383	1.702	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	44	84	158	0	0
normalized size	1	1.	0.88	0.77	1.47	2.77	0.	0.
time (sec)	N/A	0.031	0.034	0.027	1.173	1.823	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	41	22	34
normalized size	1	1.	1.	0.93	1.2	2.73	1.47	2.27
time (sec)	N/A	0.019	0.005	0.003	1.14	1.615	4.689	1.238

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	82	84	653	0	0
normalized size	1	1.	0.99	1.09	1.12	8.71	0.	0.
time (sec)	N/A	0.048	0.071	0.035	1.265	1.779	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	55	65	93	37	0
normalized size	1	1.	1.	1.62	1.91	2.74	1.09	0.
time (sec)	N/A	0.034	0.027	0.025	1.311	1.711	15.067	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	97	117	84	768	0	0
normalized size	1	1.	1.04	1.26	0.9	8.26	0.	0.
time (sec)	N/A	0.068	0.125	0.043	1.273	1.868	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	73	63	115	51	0
normalized size	1	1.	0.94	1.55	1.34	2.45	1.09	0.
time (sec)	N/A	0.059	0.043	0.035	1.184	1.651	48.208	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	1039	0	0	0	0	0
normalized size	1	1.	5.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	24.272	0.074	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	122	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.8	0.059	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	84	77	0	0	0	0
normalized size	1	1.	0.97	0.89	0.	0.	0.	0.
time (sec)	N/A	0.087	0.139	0.034	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.133	0.034	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	23	7	15
normalized size	1	1.	1.	0.88	1.	2.88	0.88	1.88
time (sec)	N/A	0.012	0.003	0.004	1.046	1.844	0.389	1.165

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	77	99	0	0	0
normalized size	1	1.	1.17	1.03	1.32	0.	0.	0.
time (sec)	N/A	0.074	0.095	0.09	1.21	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	69	99	0	0	0
normalized size	1	1.	1.17	0.92	1.32	0.	0.	0.
time (sec)	N/A	0.041	0.081	0.065	1.215	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	79	74	82	0	0	0
normalized size	1	1.	1.18	1.1	1.22	0.	0.	0.
time (sec)	N/A	0.017	0.08	0.052	1.172	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	41	178	0	0
normalized size	1	1.	0.92	1.32	1.64	7.12	0.	0.
time (sec)	N/A	0.037	0.021	0.014	1.235	1.878	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	68	77	88	0	0	0
normalized size	1	1.	0.96	1.08	1.24	0.	0.	0.
time (sec)	N/A	0.063	0.067	0.077	1.198	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	77	93	0	0	0
normalized size	1	1.	0.96	1.03	1.24	0.	0.	0.
time (sec)	N/A	0.062	0.072	0.039	1.199	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	89	0	111	0	0	0
normalized size	1	1.	0.9	0.	1.12	0.	0.	0.
time (sec)	N/A	0.137	1.435	0.07	1.224	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	111	0	0	0
normalized size	1	1.	0.86	0.	1.12	0.	0.	0.
time (sec)	N/A	0.107	1.245	0.098	1.186	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	92	0	0	0
normalized size	1	1.	0.91	0.	1.03	0.	0.	0.
time (sec)	N/A	0.065	1.075	0.066	1.163	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	40	50	217	0	0
normalized size	1	1.	0.91	0.93	1.16	5.05	0.	0.
time (sec)	N/A	0.064	0.03	0.079	1.18	1.871	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	79	0	100	0	0	0
normalized size	1	1.	0.87	0.	1.1	0.	0.	0.
time (sec)	N/A	0.128	1.375	0.062	1.218	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	161	0	201	0	0	0
normalized size	1	1.	0.97	0.	1.21	0.	0.	0.
time (sec)	N/A	0.195	1.534	0.083	1.341	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	161	0	201	0	0	0
normalized size	1	1.	0.97	0.	1.21	0.	0.	0.
time (sec)	N/A	0.133	1.556	0.072	1.271	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	140	0	169	0	0	0
normalized size	1	1.	0.93	0.	1.13	0.	0.	0.
time (sec)	N/A	0.076	1.203	0.088	1.252	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	67	84	374	0	0
normalized size	1	1.	0.78	1.	1.25	5.58	0.	0.
time (sec)	N/A	0.1	0.05	0.109	1.25	1.88	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	126	0	180	0	0	0
normalized size	1	1.	0.82	0.	1.17	0.	0.	0.
time (sec)	N/A	0.182	1.33	0.074	1.302	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	5.239	0.778	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	8.061	0.661	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	93	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.138	0.838	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	5.787	0.753	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	167	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.409	0.701	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	8.352	0.674	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	185	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.234	2.047	0.24	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	117	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	1.907	0.207	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	102	115	0	0	0	0
normalized size	1	1.	1.03	1.16	0.	0.	0.	0.
time (sec)	N/A	0.071	0.17	0.22	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	24.333	0.13	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	74	46	462	0	0
normalized size	1	1.	1.02	1.64	1.02	10.27	0.	0.
time (sec)	N/A	0.093	0.059	0.036	1.335	1.89	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	90	0	567	0	0
normalized size	1	1.	0.81	1.34	0.	8.46	0.	0.
time (sec)	N/A	0.123	0.131	0.073	0.	1.894	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	152	95	986	0	0
normalized size	1	1.	0.84	1.35	0.84	8.73	0.	0.
time (sec)	N/A	0.217	0.201	0.087	1.336	1.893	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	60	54	93	333	0	0
normalized size	1	1.	0.85	0.76	1.31	4.69	0.	0.
time (sec)	N/A	0.045	1.502	0.057	1.315	1.95	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	63	136	1192	383	0	185
normalized size	1	1.	0.56	1.2	10.55	3.39	0.	1.64
time (sec)	N/A	0.096	0.128	0.049	1.817	1.892	0.	1.283

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	44	66	948	317	0	134
normalized size	1	1.	0.81	1.22	17.56	5.87	0.	2.48
time (sec)	N/A	0.053	0.027	0.029	1.675	1.887	0.	1.155

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	36	695	144	0	53
normalized size	1	1.	0.73	0.97	18.78	3.89	0.	1.43
time (sec)	N/A	0.017	0.005	0.028	1.611	1.801	0.	1.255

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	9.856	0.036	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	13.156	0.044	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	104	831	656	251	269	2072
normalized size	1	1.	0.3	2.4	1.9	0.73	0.78	5.99
time (sec)	N/A	0.418	1.296	0.01	1.095	2.091	2.444	1.906

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	72	303	396	169	151	780
normalized size	1	1.	0.43	1.81	2.37	1.01	0.9	4.67
time (sec)	N/A	0.186	0.199	0.01	1.22	2.036	0.771	1.575

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	63	150	112	65	281
normalized size	1	1.	0.93	1.17	2.78	2.07	1.2	5.2
time (sec)	N/A	0.046	0.063	0.007	1.148	2.037	0.628	1.324

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	130	0	0	547	0	0
normalized size	1	1.	1.05	0.	0.	4.41	0.	0.
time (sec)	N/A	0.287	0.964	0.016	0.	2.156	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	199	0	0	757	0	0
normalized size	1	1.	1.09	0.	0.	4.16	0.	0.
time (sec)	N/A	0.361	3.042	0.016	0.	2.147	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	378	1815	867	467	0	2919
normalized size	1	1.	0.7	3.38	1.61	0.87	0.	5.44
time (sec)	N/A	0.697	2.874	0.01	1.183	2.058	0.	3.707

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	118	659	501	294	0	953
normalized size	1	1.	0.45	2.52	1.92	1.13	0.	3.65
time (sec)	N/A	0.315	0.344	0.007	1.23	2.09	0.	2.256

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	133	185	159	94	173
normalized size	1	1.	0.76	1.56	2.18	1.87	1.11	2.04
time (sec)	N/A	0.079	0.082	0.007	1.156	2.055	1.738	1.687

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	233	0	0	1539	0	0
normalized size	1	1.	1.	0.	0.	6.63	0.	0.
time (sec)	N/A	0.519	0.068	0.016	0.	2.422	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	210	0	0	2032	0	0
normalized size	1	1.	0.64	0.	0.	6.18	0.	0.
time (sec)	N/A	0.724	1.836	0.015	0.	2.5	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [88] had the largest ratio of [0.75]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.	12	0.25
2	A	4	4	1.	12	0.333
3	A	2	2	1.	10	0.2
4	A	3	3	1.	8	0.375
5	A	3	3	1.	12	0.25
6	A	4	4	1.	12	0.333
7	A	5	5	1.	12	0.417
8	A	3	3	1.	14	0.214
9	A	6	5	1.	14	0.357
10	A	3	3	1.	12	0.25
11	A	5	4	1.	10	0.4
12	A	5	4	1.	14	0.286
13	A	6	6	1.	14	0.429
14	A	7	6	1.	14	0.429
15	A	4	4	1.	14	0.286
16	A	10	5	1.	14	0.357
17	A	3	2	1.	12	0.167
18	A	8	4	1.	10	0.4
19	A	8	4	1.	14	0.286
20	A	9	5	1.	14	0.357
21	A	12	6	1.	14	0.429
22	A	3	2	1.	12	0.167
23	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	8	3	1.	16	0.188
25	A	5	3	1.	16	0.188
26	A	3	2	1.	14	0.143
27	A	0	0	0.	0	0.
28	A	2	2	1.	12	0.167
29	A	7	5	1.	12	0.417
30	A	6	5	1.	10	0.5
31	A	5	5	1.	8	0.625
32	A	3	3	1.	12	0.25
33	A	2	2	1.	12	0.167
34	A	3	3	1.	12	0.25
35	A	4	3	1.	12	0.25
36	A	5	3	1.	12	0.25
37	A	9	4	1.	16	0.25
38	A	6	4	1.	16	0.25
39	A	4	3	1.	14	0.214
40	A	0	0	0.	0	0.
41	A	7	6	1.	12	0.5
42	A	6	5	1.	12	0.417
43	A	6	6	1.	12	0.5
44	A	5	5	1.	10	0.5
45	A	5	5	1.	8	0.625
46	A	3	3	1.	12	0.25
47	A	4	4	1.	12	0.333
48	A	2	2	1.	12	0.167
49	A	5	5	1.	12	0.417
50	A	3	3	1.	12	0.25
51	A	6	6	1.	12	0.5
52	A	4	3	1.	12	0.25
53	A	9	4	1.	16	0.25
54	A	6	4	1.	16	0.25
55	A	4	3	1.	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	0	0	0.	0	0.
57	A	2	2	1.	12	0.167
58	A	3	2	1.	12	0.167
59	A	3	2	1.	10	0.2
60	A	3	2	1.	8	0.25
61	A	3	3	1.	12	0.25
62	A	3	2	1.	12	0.167
63	A	3	2	1.	12	0.167
64	A	5	3	1.	14	0.214
65	A	5	3	1.	12	0.25
66	A	5	3	1.	10	0.3
67	A	5	4	1.	14	0.286
68	A	5	3	1.	14	0.214
69	A	8	3	1.	14	0.214
70	A	8	3	1.	12	0.25
71	A	8	3	1.	10	0.3
72	A	8	4	1.	14	0.286
73	A	8	3	1.	14	0.214
74	A	0	0	0.	0	0.
75	A	0	0	0.	0	0.
76	A	3	3	1.	20	0.15
77	A	0	0	0.	0	0.
78	A	5	5	1.	22	0.227
79	A	0	0	0.	0	0.
80	A	8	3	1.	16	0.188
81	A	5	3	1.	16	0.188
82	A	3	2	1.	14	0.143
83	A	0	0	0.	0	0.
84	A	5	5	1.	16	0.312
85	A	7	6	1.	18	0.333
86	A	12	6	1.	18	0.333
87	A	4	4	1.	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	12	9	1.	12	0.75
89	A	8	7	1.	10	0.7
90	A	4	4	1.	8	0.5
91	A	0	0	0.	0	0.
92	A	0	0	0.	0	0.
93	A	16	4	1.	18	0.222
94	A	10	4	1.	16	0.25
95	A	4	4	1.	14	0.286
96	A	10	5	1.	18	0.278
97	A	11	6	1.	18	0.333
98	A	23	6	1.	18	0.333
99	A	13	5	1.	16	0.312
100	A	5	4	1.	14	0.286
101	A	13	5	1.	18	0.278
102	A	14	6	1.	18	0.333

Chapter 3

Listing of integrals

3.1 $\int x^3 \sinh\left(a+bx^2\right) dx$

Optimal. Leaf size=34

$$\frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh\left(a+bx^2\right)}{2b^2}$$

```
[Out] (x^2*Cosh[a + b*x^2])/(2*b) - Sinh[a + b*x^2]/(2*b^2)
```

Rubi [A] time = 0.0355861, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 3296, 2637}

$$\frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^2],x]
```

```
[Out] (x^2*Cosh[a + b*x^2])/(2*b) - Sinh[a + b*x^2]/(2*b^2)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
```

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \sinh\left(a+bx^2\right) dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sinh(a+bx)\,dx, x, x^2\right) \\
&= \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\operatorname{Subst}\left(\int \cosh(a+bx)\,dx, x, x^2\right)}{2b} \\
&= \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\sinh\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.0344656, size = 31, normalized size = 0.91

$$
\frac{bx^2\cosh\left(a+bx^2\right)-\sinh\left(a+bx^2\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b*x^2] - Sinh[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.013, size = 45, normalized size = 1.3

$$
\frac{\left(bx^2-1\right)e^{bx^2+a}}{4\,b^2}+\frac{\left(bx^2+1\right)e^{-bx^2-a}}{4\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^2+a),x)
```

```
[Out] 1/4*(b*x^2-1)/b^2*exp(b*x^2+a)+1/4*(b*x^2+1)/b^2*exp(-b*x^2-a)
```

Maxima [B] time = 1.03451, size = 109, normalized size = 3.21

$$\frac{1}{4}x^4\sinh\left(bx^2+a\right)-\frac{1}{8}b\left(\frac{\left(b^2x^4e^a-2\,bx^2e^a+2\,e^a\right)e^{\left(bx^2\right)}}{b^3}-\frac{\left(b^2x^4+2\,bx^2+2\right)e^{\left(-bx^2-a\right)}}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*sinh(b*x^2 + a) - 1/8*b*((b^2*x^4*e^a - 2*b*x^2*e^a + 2*e^a)*e^(b*x
^2)/b^3 - (b^2*x^4 + 2*b*x^2 + 2)*e^(-b*x^2 - a)/b^3)
```

Fricas [A] time = 1.98473, size = 69, normalized size = 2.03

$$\frac{bx^2\cosh\left(bx^2+a\right)-\sinh\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*cosh(b*x^2 + a) - sinh(b*x^2 + a))/b^2
```

Sympy [A] time = 1.12747, size = 36, normalized size = 1.06

$$\begin{cases}\frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh\left(a+bx^2\right)}{2b^2} & \text{for } b\neq 0\\ \frac{x^4\sinh\left(a\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**2+a),x)
```

```
[Out] Piecewise((x**2*cosh(a + b*x**2)/(2*b) - sinh(a + b*x**2)/(2*b**2), Ne(b, 0
)), (x**4*sinh(a)/4, True))
```

Giac [A] time = 1.39141, size = 63, normalized size = 1.85

$$\frac{\frac{(bx^2-1)e^{(bx^2+a)}}{b}+\frac{(bx^2+1)e^{(-bx^2-a)}}{b}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 - 1)*e^(b*x^2 + a)/b + (b*x^2 + 1)*e^(-b*x^2 - a)/b)/b
```

3.2 $\int x^2 \sinh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x\right)}{8b^{3/2}}-\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\sqrt{b}x\right)}{8b^{3/2}}+\frac{x\cosh\left(a+bx^2\right)}{2b}$$

```
[Out] (x*Cosh[a + b*x^2])/(2*b) - (Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*b^(3/2)*E^a) - (E^
a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(8*b^(3/2))
```

Rubi [A] time = 0.0414219, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5324, 5299, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x\right)}{8b^{3/2}}-\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\sqrt{b}x\right)}{8b^{3/2}}+\frac{x\cosh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^2],x]
```

```
[Out] (x*Cosh[a + b*x^2])/(2*b) - (Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*b^(3/2)*E^a) - (E^
a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(8*b^(3/2))
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^2\sinh\left(a+bx^2\right)dx &= \frac{x\cosh\left(a+bx^2\right)}{2b}-\frac{\int\cosh\left(a+bx^2\right)dx}{2b}\\ &= \frac{x\cosh\left(a+bx^2\right)}{2b}-\frac{\int e^{-a-bx^2}dx}{4b}-\frac{\int e^{a+bx^2}dx}{4b}\\ &= \frac{x\cosh\left(a+bx^2\right)}{2b}-\frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)}{8b^{3/2}}-\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0682744, size = 67, normalized size = 0.97

$$\frac{\sqrt{\pi}(\sinh(a)-\cosh(a))\mathrm{Erf}\left(\sqrt{b}x\right)-\sqrt{\pi}(\sinh(a)+\cosh(a))\mathrm{Erfi}\left(\sqrt{b}x\right)+4\sqrt{b}x\cosh\left(a+bx^2\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^2],x]
```

```
[Out] (4*Sqrt[b]*x*Cosh[a + b*x^2] + Sqrt[Pi]*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a])
 - Sqrt[Pi]*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]))/(8*b^(3/2))
```

Maple [A] time = 0.048, size = 74, normalized size = 1.1

$$\frac{e^{-a}xe^{-bx^2}}{4b}-\frac{e^{-a}\sqrt{\pi}}{8}\mathrm{Erf}\left(x\sqrt{b}\right)b^{-\frac{3}{2}}+\frac{e^{a}e^{bx^2}x}{4b}-\frac{e^{a}\sqrt{\pi}}{8b}\mathrm{Erf}\left(\sqrt{-b}x\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(b*x^2+a),x)
```

```
[Out] 1/4*exp(-a)/b*x*exp(-b*x^2)-1/8*exp(-a)/b^(3/2)*Pi^(1/2)*erf(x*b^(1/2))+1/4
*exp(a)*exp(b*x^2)*x/b-1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [B] time = 0.992813, size = 149, normalized size = 2.16

$$\frac{1}{3}x^3\sinh\left(bx^2+a\right)-\frac{1}{24}b\left(\frac{2\left(2bx^3e^a-3xe^a\right)e^{\left(bx^2\right)}}{b^2}-\frac{2\left(2bx^3+3x\right)e^{\left(-bx^2-a\right)}}{b^2}+\frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{b^{\frac{5}{2}}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right.}{\sqrt{-b}b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(b*x^2 + a) - 1/24*b*(2*(2*b*x^3*e^a - 3*x*e^a)*e^(b*x^2)/b^2 -
 2*(2*b*x^3 + 3*x)*e^(-b*x^2 - a)/b^2 + 3*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^
(5/2) + 3*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b^2))
```

Fricas [B] time = 2.11412, size = 539, normalized size = 7.81

$$\frac{2bx\cosh\left(bx^2+a\right)^2+4bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)+2bx\sinh\left(bx^2+a\right)^2+\sqrt{\pi}\left(\cosh\left(bx^2+a\right)\cosh\left(a\right)+(\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x*cosh(b*x^2 + a)^2 + 4*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + 2*b*
x*sinh(b*x^2 + a)^2 + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) + sinh(a
))*sinh(b*x^2 + a) + cosh(b*x^2 + a)*sinh(a))*sqrt(-b)*erf(sqrt(-b)*x) - sq
rt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a) - cos
h(b*x^2 + a)*sinh(a))*sqrt(b)*erf(sqrt(b)*x) + 2*b*x)/(b^2*cosh(b*x^2 + a)
+ b^2*sinh(b*x^2 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sinh\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(b*x**2+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2), x)
```

Giac [A] time = 1.30708, size = 101, normalized size = 1.46

$$\frac{xe^{\left(bx^2+a\right)}}{4\,b}+\frac{xe^{\left(-bx^2-a\right)}}{4\,b}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{8\,b^{\frac{3}{2}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}x\right)e^{a}}{8\,\sqrt{-b}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*x*e^(b*x^2 + a)/b + 1/4*x*e^(-b*x^2 - a)/b + 1/8*sqrt(pi)*erf(-sqrt(b)*
x)*e^(-a)/b^(3/2) + 1/8*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

3.3 $\int x \sinh\left(a + bx^2\right) dx$

Optimal. Leaf size=15

$$\frac{\cosh\left(a + bx^2\right)}{2b}$$

```
[Out] Cosh[a + b*x^2]/(2*b)
```

Rubi [A] time = 0.0161585, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5320, 2638}

$$\frac{\cosh\left(a + bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2],x]
```

```
[Out] Cosh[a + b*x^2]/(2*b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x \sinh\left(a+bx^2\right) dx = \frac{1}{2}\,\text{Subst}\left(\int \sinh(a+bx)\,dx, x, x^2\right)$$
$$= \frac{\cosh\left(a+bx^2\right)}{2b}$$

Mathematica [B] time = 0.0098784, size = 31, normalized size = 2.07

$$\frac{\sinh(a)\sinh\left(bx^2\right)}{2b} + \frac{\cosh(a)\cosh\left(bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2],x]
```

```
[Out] (Cosh[a]*Cosh[b*x^2])/(2*b) + (Sinh[a]*Sinh[b*x^2])/(2*b)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\cosh\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a),x)
```

```
[Out] 1/2*cosh(b*x^2+a)/b
```

Maxima [A] time = 1.13553, size = 18, normalized size = 1.2

$$\frac{\cosh\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(b*x^2 + a)/b
```

Fricas [A] time = 1.99599, size = 31, normalized size = 2.07

$$\frac{\cosh\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*cosh(b*x^2 + a)/b
```

Sympy [A] time = 0.258525, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\cosh\left(a+bx^2\right)}{2b} & \text{for}\ b \neq 0 \\ \frac{x^2 \sinh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a),x)
```

```
[Out] Piecewise((cosh(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*sinh(a)/2, True))
```

Giac [A] time = 1.25151, size = 34, normalized size = 2.27

$$\frac{e^{\left(bx^2+a\right)} + e^{\left(-bx^2-a\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(e^(b*x^2 + a) + e^(-b*x^2 - a))/b
```

3.4 $\int \sinh\left(a+bx^2\right)dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\pi}e^a\text{Erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[b]*x])/(4*Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])
/(4*Sqrt[b])
```

Rubi [A] time = 0.0182759, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5298, 2204, 2205}

$$\frac{\sqrt{\pi}e^a\text{Erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[b]*x])/(4*Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])
/(4*Sqrt[b])
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sinh\left(a+bx^2\right)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^2}\,dx\right)+\frac{1}{2}\int e^{a+bx^2}\,dx$$
$$= -\frac{e^{-a}\sqrt{\pi}\mathrm{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\mathrm{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}$$

Mathematica [A] time = 0.0384149, size = 45, normalized size = 0.85

$$\frac{\sqrt{\pi}\left((\sinh(a)-\cosh(a))\mathrm{Erf}\left(\sqrt{b}x\right)+(\sinh(a)+\cosh(a))\mathrm{Erfi}\left(\sqrt{b}x\right)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Erfi[Sqrt[b]*x]*(Cosh[a] +
 Sinh[a])))/(4*Sqrt[b])
```

Maple [A] time = 0.017, size = 40, normalized size = 0.8

$$-\frac{\sqrt{\pi}\mathrm{e}^{-a}}{4}\mathrm{Erf}\left(x\sqrt{b}\right)\frac{1}{\sqrt{b}}+\frac{\mathrm{e}^{a}\sqrt{\pi}}{4}\mathrm{Erf}\left(\sqrt{-b}x\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a),x)
```

```
[Out] -1/4*erf(x*b^(1/2))*Pi^(1/2)*exp(-a)/b^(1/2)+1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)
*erf((-b)^(1/2)*x)
```

Maxima [B] time = 1.14263, size = 116, normalized size = 2.19

$$-\frac{1}{4}b\left(\frac{2\,xe^{\left(bx^2+a\right)}}{b}-\frac{2\,xe^{\left(-bx^2-a\right)}}{b}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)e^{(-a)}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}x\right)e^{a}}{\sqrt{-b}b}\right)+x\sinh\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*x*e^(b*x^2 + a)/b - 2*x*e^(-b*x^2 - a)/b + sqrt(pi)*erf(sqrt(b)*x
)*e^(-a)/b^(3/2) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)) + x*sinh(b*x^
2 + a)
```

Fricas [A] time = 1.91211, size = 159, normalized size = 3.

$$-\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\operatorname{erf}\left(\sqrt{-b}x\right)+\sqrt{\pi}\sqrt{b}(\cosh(a)-\sinh(a))\operatorname{erf}\left(\sqrt{b}x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) + sqrt(pi)*sqrt
(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a),x)
```

```
[Out] Integral(sinh(a + b*x**2), x)
```

Giac [A] time = 1.20513, size = 55, normalized size = 1.04

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{4\sqrt{b}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}x\right)e^{a}}{4\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 1/4*sqrt(pi)*erf(-sqrt(-b)*x)
*e^a/sqrt(-b)
```

3.5 $\int \frac{\sinh\left(a+bx^2\right)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sinh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\cosh(a)\text{Shi}\left(bx^2\right)$$

```
[Out] (CoshIntegral[b*x^2]*Sinh[a])/2 + (Cosh[a]*SinhIntegral[b*x^2])/2
```

Rubi [A] time = 0.0353859, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5318, 5317, 5316}

$$\frac{1}{2}\sinh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\cosh(a)\text{Shi}\left(bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]/x,x]
```

```
[Out] (CoshIntegral[b*x^2]*Sinh[a])/2 + (Cosh[a]*SinhIntegral[b*x^2])/2
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sinh(a+bx^2)}{x}\,dx = \cosh(a)\int \frac{\sinh(bx^2)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx^2)}{x}\,dx$$
$$= \frac{1}{2}\mathbf{Chi}\left(bx^2\right)\sinh(a) + \frac{1}{2}\cosh(a)\mathbf{Shi}\left(bx^2\right)$$

Mathematica [A] time = 0.0135722, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(\sinh(a)\mathbf{Chi}\left(bx^2\right) + \cosh(a)\mathbf{Shi}\left(bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]/x,x]
```

```
[Out] (CoshIntegral[b*x^2]*Sinh[a] + Cosh[a]*SinhIntegral[b*x^2])/2
```

Maple [A] time = 0.012, size = 27, normalized size = 1.1

$$\frac{e^{-a}\mathrm{Ei}\left(1, bx^2\right)}{4} - \frac{e^{a}\mathrm{Ei}\left(1, -bx^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)/x,x)
```

```
[Out] 1/4*exp(-a)*Ei(1,b*x^2)-1/4*exp(a)*Ei(1,-b*x^2)
```

Maxima [A] time = 1.25246, size = 32, normalized size = 1.28

$$-\frac{1}{4}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \frac{1}{4}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] -1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

Fricas [A] time = 1.58646, size = 104, normalized size = 4.16

$$\frac{1}{4}\left(\mathrm{Ei}\left(bx^2\right) - \mathrm{Ei}\left(-bx^2\right)\right)\cosh\left(a\right) + \frac{1}{4}\left(\mathrm{Ei}\left(bx^2\right) + \mathrm{Ei}\left(-bx^2\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(Ei(b*x^2) - Ei(-b*x^2))*cosh(a) + 1/4*(Ei(b*x^2) + Ei(-b*x^2))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x**2)/x, x)
```

Giac [A] time = 1.11746, size = 32, normalized size = 1.28

$$-\frac{1}{4}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \frac{1}{4}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] -1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

3.6 $\int \frac{\sinh(a+bx^2)}{x^2} dx$

Optimal. Leaf size=66

$$\frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\text{Erf}\left(\sqrt{b}x\right)+\frac{1}{2}\sqrt{\pi}e^{a}\sqrt{b}\text{Erfi}\left(\sqrt{b}x\right)-\frac{\sinh\left(a+bx^2\right)}{x}$$

```
[Out] (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(2*E^a) + (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt
[b]*x])/2 - Sinh[a + b*x^2]/x
```

Rubi [A] time = 0.0345971, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5326, 5299, 2204, 2205}

$$\frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\text{Erf}\left(\sqrt{b}x\right)+\frac{1}{2}\sqrt{\pi}e^{a}\sqrt{b}\text{Erfi}\left(\sqrt{b}x\right)-\frac{\sinh\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(2*E^a) + (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt
[b]*x])/2 - Sinh[a + b*x^2]/x
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sinh\left(a+bx^2\right)}{x}+(2b)\int \cosh\left(a+bx^2\right)\,dx\\ &= -\frac{\sinh\left(a+bx^2\right)}{x}+b\int e^{-a-bx^2}\,dx+b\int e^{a+bx^2}\,dx\\ &= \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)+\frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)-\frac{\sinh\left(a+bx^2\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.063534, size = 70, normalized size = 1.06

$$\frac{\sqrt{\pi}\sqrt{b}x(\cosh(a)-\sinh(a))\mathrm{Erf}\left(\sqrt{b}x\right)+\sqrt{\pi}\sqrt{b}x(\sinh(a)+\cosh(a))\mathrm{Erfi}\left(\sqrt{b}x\right)-2\sinh\left(a+bx^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + Sqrt[b]*Sqrt[Pi]*x
*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]) - 2*Sinh[a + b*x^2])/(2*x)
```

Maple [A] time = 0.023, size = 70, normalized size = 1.1

$$\frac{e^{-a}e^{-bx^2}}{2x}+\frac{e^{-a}\sqrt{\pi}}{2}\sqrt{b}\mathrm{Erf}\left(x\sqrt{b}\right)-\frac{e^{a}e^{bx^2}}{2x}+\frac{e^{a}b\sqrt{\pi}}{2}\mathrm{Erf}\left(\sqrt{-b}x\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)/x^2,x)
```

```
[Out] 1/2*exp(-a)/x*exp(-b*x^2)+1/2*exp(-a)*b^(1/2)*Pi^(1/2)*erf(x*b^(1/2))-1/2*e
xp(a)*exp(b*x^2)/x+1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [A] time = 1.11713, size = 73, normalized size = 1.11

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{\sqrt{b}}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right)e^{a}}{\sqrt{-b}}\right)b-\frac{\sinh\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) + sqrt(pi)*erf(sqrt(-b)*x)*e^a/
sqrt(-b))*b - sinh(b*x^2 + a)/x
```

Fricas [B] time = 1.81125, size = 529, normalized size = 8.02

$$-\frac{\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)\cosh(a)+x\cosh\left(bx^2+a\right)\sinh(a)+(x\cosh(a)+x\sinh(a))\sinh\left(bx^2+a\right)\right)\sqrt{-b}\operatorname{erf}\left(\sqrt{-b}x\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a) + (x*
cosh(a) + x*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) - sqrt(pi)*(
x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a) + (x*cosh(a) - x*sinh
(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + cosh(b*x^2 + a)^2 + 2*cosh(b
*x^2 + a)*sinh(b*x^2 + a) + sinh(b*x^2 + a)^2 - 1)/(x*cosh(b*x^2 + a) + x*s
inh(b*x^2 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^2 + a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^2 + a)/x^2, x)
```

3.7 $\int \frac{\sinh(a+bx^2)}{x^3} dx$

Optimal. Leaf size=42

$$\frac{1}{2}b\cosh(a)\mathbf{Chi}\left(bx^2\right)+\frac{1}{2}b\sinh(a)\mathbf{Shi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{2x^2}$$

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2])/2 - Sinh[a + b*x^2]/(2*x^2) + (b*Sinh[a]*Si
nhIntegral[b*x^2])/2
```

Rubi [A] time = 0.0922669, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5320, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\cosh(a)\mathbf{Chi}\left(bx^2\right)+\frac{1}{2}b\sinh(a)\mathbf{Shi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]/x^3,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2])/2 - Sinh[a + b*x^2]/(2*x^2) + (b*Sinh[a]*Si
nhIntegral[b*x^2])/2
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sinh\left(a+bx^2\right)}{2x^2}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, x^2\right)\\
&= -\frac{\sinh\left(a+bx^2\right)}{2x^2}+\frac{1}{2}(b\cosh(a))\operatorname{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, x^2\right)+\frac{1}{2}(b\sinh(a))\operatorname{Subst}\Bigg(\int \frac{\sinh(bx}{x}\\
&= \frac{1}{2}b\cosh(a)\operatorname{Chi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{2x^2}+\frac{1}{2}b\sinh(a)\operatorname{Shi}\left(bx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0398667, size = 38, normalized size = 0.9

$$
\frac{1}{2}\left(b\cosh(a)\operatorname{Chi}\left(bx^2\right)+b\sinh(a)\operatorname{Shi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]/x^3,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2] - Sinh[a + b*x^2]/x^2 + b*Sinh[a]*SinhIntegr
al[b*x^2])/2
```

Maple [A] time = 0.017, size = 58, normalized size = 1.4

$$\frac{e^{-a}e^{-bx^2}}{4x^2}-\frac{e^{-a}b\mathrm{Ei}\left(1,bx^2\right)}{4}-\frac{e^{a}e^{bx^2}}{4x^2}-\frac{e^{a}b\mathrm{Ei}\left(1,-bx^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)/x^3,x)
```

```
[Out] 1/4*exp(-a)/x^2*exp(-b*x^2)-1/4*exp(-a)*b*Ei(1,b*x^2)-1/4*exp(a)*exp(b*x^2)
/x^2-1/4*exp(a)*b*Ei(1,-b*x^2)
```

Maxima [A] time = 1.27355, size = 53, normalized size = 1.26

$$\frac{1}{4}\left(\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+\mathrm{Ei}\left(bx^2\right)e^{a}\right)b-\frac{\sinh\left(bx^2+a\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(Ei(-b*x^2)*e^(-a) + Ei(b*x^2)*e^a)*b - 1/2*sinh(b*x^2 + a)/x^2
```

Fricas [A] time = 1.68495, size = 166, normalized size = 3.95

$$\frac{\left(bx^2\mathrm{Ei}\left(bx^2\right)+bx^2\mathrm{Ei}\left(-bx^2\right)\right)\cosh\left(a\right)+\left(bx^2\mathrm{Ei}\left(bx^2\right)-bx^2\mathrm{Ei}\left(-bx^2\right)\right)\sinh\left(a\right)-2\sinh\left(bx^2+a\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*cosh(a) + (b*x^2*Ei(b*x^2) - b*x^
2*Ei(-b*x^2))*sinh(a) - 2*sinh(b*x^2 + a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)/x**3, x)
```

Giac [B] time = 1.15456, size = 147, normalized size = 3.5

$$\frac{\left(bx^2 + a\right)b^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)} - ab^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \left(bx^2 + a\right)b^2\mathrm{Ei}\left(bx^2\right)e^{a} - ab^2\mathrm{Ei}\left(bx^2\right)e^{a} - b^2e^{\left(bx^2+a\right)} + b^2e^{\left(-bx^2-a\right)}}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) - a*b^2*Ei(-b*x^2)*e^(-a) + (b*x^2 +
 a)*b^2*Ei(b*x^2)*e^a - a*b^2*Ei(b*x^2)*e^a - b^2*e^(b*x^2 + a) + b^2*e^(-b
*x^2 - a))/(b^2*x^2)
```

3.8 $\int x^3 \sinh^2\left(a + bx^2\right) dx$

Optimal. Leaf size=51

$$-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^4}{8}$$

```
[Out] -x^4/8 + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b) - Sinh[a + b*x^2]^2/(8
*b^2)
```

Rubi [A] time = 0.0499978, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5320, 3310, 30}

$$-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^2]^2,x]
```

```
[Out] -x^4/8 + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b) - Sinh[a + b*x^2]^2/(8
*b^2)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3 \sinh^2\left(a+bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sinh^2(a+bx)\,dx, x, x^2\right)\\ &= \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} - \frac{\sinh^2\left(a+bx^2\right)}{8b^2} - \frac{1}{4}\operatorname{Subst}\left(\int x\,dx, x, x^2\right)\\ &= -\frac{x^4}{8} + \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} - \frac{\sinh^2\left(a+bx^2\right)}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.101511, size = 42, normalized size = 0.82

$$-\frac{2bx^2\left(bx^2-\sinh\left(2\left(a+bx^2\right)\right)\right)+\cosh\left(2\left(a+bx^2\right)\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^2]^2,x]
```

```
[Out] -(Cosh[2*(a + b*x^2)] + 2*b*x^2*(b*x^2 - Sinh[2*(a + b*x^2)]))/(16*b^2)
```

Maple [A] time = 0.029, size = 55, normalized size = 1.1

$$-\frac{x^4}{8} + \frac{\left(2\,bx^2-1\right)e^{2\,bx^2+2\,a}}{32\,b^2} - \frac{\left(2\,bx^2+1\right)e^{-2\,bx^2-2\,a}}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^2+a)^2,x)
```

```
[Out] -1/8*x^4+1/32*(2*b*x^2-1)/b^2*exp(2*b*x^2+2*a)-1/32*(2*b*x^2+1)/b^2*exp(-2*
b*x^2-2*a)
```

Maxima [A] time = 1.0896, size = 80, normalized size = 1.57

$$-\frac{1}{8}x^4+\frac{\left(2\,bx^2e^{(2\,a)}-e^{(2\,a)}\right)e^{\left(2\,bx^2\right)}}{32\,b^2}-\frac{\left(2\,bx^2+1\right)e^{\left(-2\,bx^2-2\,a\right)}}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*x^4 + 1/32*(2*b*x^2*e^(2*a) - e^(2*a))*e^(2*b*x^2)/b^2 - 1/32*(2*b*x^2
 + 1)*e^(-2*b*x^2 - 2*a)/b^2
```

Fricas [A] time = 1.67135, size = 142, normalized size = 2.78

$$-\frac{2\,b^2x^4-4\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)+\cosh\left(bx^2+a\right)^2+\sinh\left(bx^2+a\right)^2}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^2*x^4 - 4*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + cosh(b*x^2 + a
)^2 + sinh(b*x^2 + a)^2)/b^2
```

Sympy [A] time = 2.55508, size = 78, normalized size = 1.53

$$\begin{cases}\frac{x^4\sinh^2\left(a+bx^2\right)}{8}-\frac{x^4\cosh^2\left(a+bx^2\right)}{8}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{\sinh^2\left(a+bx^2\right)}{8b^2} & \text{for}\ b\neq 0\\ \frac{x^4\sinh^2(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**4*sinh(a + b*x**2)**2/8 - x**4*cosh(a + b*x**2)**2/8 + x**2*s
inh(a + b*x**2)*cosh(a + b*x**2)/(4*b) - sinh(a + b*x**2)**2/(8*b**2), Ne(b
, 0)), (x**4*sinh(a)**2/4, True))
```

Giac [B] time = 1.17557, size = 158, normalized size = 3.1

$$-\frac{4\left(bx^2+a\right)^2-8\left(bx^2+a\right)a-2\left(bx^2+a\right)e^{\left(2\,bx^2+2\,a\right)}+2\,ae^{\left(2\,bx^2+2\,a\right)}+2\left(bx^2+a\right)e^{\left(-2\,bx^2-2\,a\right)}-2\,ae^{\left(-2\,bx^2-2\,a\right)}+e^{\left(2\,bx^2+2\,a\right)}+\cdots}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*(b*x^2 + a)^2 - 8*(b*x^2 + a)*a - 2*(b*x^2 + a)*e^(2*b*x^2 + 2*a)
+ 2*a*e^(2*b*x^2 + 2*a) + 2*(b*x^2 + a)*e^(-2*b*x^2 - 2*a) - 2*a*e^(-2*b*x^
2 - 2*a) + e^(2*b*x^2 + 2*a) + e^(-2*b*x^2 - 2*a))/b^2
```

3.9 $\int x^2 \sinh^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{\frac{\pi}{2}}e^{-2a}\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}-\frac{x^3}{6}$$

```
[Out] -x^3/6 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)*E^(2*a)) - (E^(2*a
)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)) + (x*Sinh[2*a + 2*b*x^2]
)/(8*b)
```

Rubi [A] time = 0.0956949, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5340, 5325, 5298, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{-2a}\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}-\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^2]^2,x]
```

```
[Out] -x^3/6 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)*E^(2*a)) - (E^(2*a
)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)) + (x*Sinh[2*a + 2*b*x^2]
)/(8*b)
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh^2\left(a+bx^2\right)\,dx &= \int \left(-\frac{x^2}{2}+\frac{1}{2}x^2\cosh\left(2a+2bx^2\right)\right)dx \\
&= -\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh\left(2a+2bx^2\right)\,dx \\
&= -\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}-\frac{\int \sinh\left(2a+2bx^2\right)\,dx}{8b} \\
&= -\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}+\frac{\int e^{-2a-2bx^2}\,dx}{16b}-\frac{\int e^{2a+2bx^2}\,dx}{16b} \\
&= -\frac{x^3}{6}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}-\frac{e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.224084, size = 101, normalized size = 1.02

$$\frac{3\sqrt{2\pi}(\cosh(2a)-\sinh(2a))\mathrm{Erf}\left(\sqrt{2}\sqrt{b}x\right)-3\sqrt{2\pi}(\sinh(2a)+\cosh(2a))\mathrm{Erfi}\left(\sqrt{2}\sqrt{b}x\right)+8\sqrt{b}x\left(3\sinh\left(2\left(a+bx^2\right)\right)-4b\right.}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^2]^2,x]
```

```
[Out] (3*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*a]) - 3*Sqrt[2*Pi]
*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) + 8*Sqrt[b]*x*(-4*b*x^2 +
```

```
3*Sinh[2*(a + b*x^2)]))/(192*b^(3/2))
```

Maple [A] time = 0.056, size = 90, normalized size = 0.9

$$-\frac{x^3}{6}-\frac{e^{-2a}xe^{-2bx^2}}{16b}+\frac{e^{-2a}\sqrt{\pi}\sqrt{2}}{64}\mathrm{Erf}\left(x\sqrt{2}\sqrt{b}\right)b^{-\frac{3}{2}}+\frac{e^{2a}xe^{2bx^2}}{16b}-\frac{e^{2a}\sqrt{\pi}}{32b}\mathrm{Erf}\left(\sqrt{-2b}x\right)\frac{1}{\sqrt{-2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(b*x^2+a)^2,x)
```

```
[Out] -1/6*x^3-1/16*exp(-2*a)/b*x*exp(-2*b*x^2)+1/64*exp(-2*a)/b^(3/2)*Pi^(1/2)*2
^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/16*exp(2*a)/b*x*exp(2*b*x^2)-1/32*exp(2*a)/
b*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Maxima [A] time = 1.72657, size = 128, normalized size = 1.29

$$-\frac{1}{6}x^3-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{64\sqrt{-b}b}+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{64b^{\frac{3}{2}}}+\frac{xe^{\left(2bx^2+2a\right)}}{16b}-\frac{xe^{\left(-2bx^2-2a\right)}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^3 - 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*
b) + 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*x
*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

Fricas [B] time = 1.76192, size = 1126, normalized size = 11.37

$$-\frac{32\,b^2x^3\cosh\left(bx^2+a\right)^2-12\,bx\cosh\left(bx^2+a\right)^4-48\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3-12\,bx\sinh\left(bx^2+a\right)^4-3\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(32*b^2*x^3*cosh(b*x^2 + a)^2 - 12*b*x*cosh(b*x^2 + a)^4 - 48*b*x*co
sh(b*x^2 + a)*sinh(b*x^2 + a)^3 - 12*b*x*sinh(b*x^2 + a)^4 - 3*sqrt(2)*sqrt
(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) + sinh(2*a))*sinh(b*x^2 + a)
^2 + cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) + cosh(b*x^
2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(2)*sqrt(-b)*x) - 3*sqr
t(2)*sqrt(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) - sinh(2*a))*sinh(b
*x^2 + a)^2 - cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) -
cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x)
+ 8*(4*b^2*x^3 - 9*b*x*cosh(b*x^2 + a)^2)*sinh(b*x^2 + a)^2 + 12*b*x + 16*(
4*b^2*x^3*cosh(b*x^2 + a) - 3*b*x*cosh(b*x^2 + a)^3)*sinh(b*x^2 + a))/(b^2*
cosh(b*x^2 + a)^2 + 2*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + b^2*sinh(b*x^2
+ a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh^2\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2)**2, x)
```

Giac [A] time = 1.16972, size = 131, normalized size = 1.32

$$-\frac{1}{6}x^3 + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{64\sqrt{-b}b} - \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{64\,b^{\frac{3}{2}}} + \frac{xe^{\left(2bx^2+2a\right)}}{16\,b} - \frac{xe^{\left(-2bx^2-2a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*x^3 + 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)
*b) - 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16
*x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

3.10 $\int x \sinh^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=31

$$\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^2}{4}$$

```
[Out] -x^2/4 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)
```

Rubi [A] time = 0.0274584, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 2635, 8}

$$\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2]^2,x]
```

```
[Out] -x^2/4 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x\sinh^2\left(a+bx^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sinh^2(a+bx)\,dx,x,x^2\right)\\ &= \frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}-\frac{1}{4}\,\text{Subst}\left(\int 1\,dx,x,x^2\right)\\ &= -\frac{x^2}{4}+\frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0242137, size = 27, normalized size = 0.87

$$\frac{\sinh\left(2\left(a+bx^2\right)\right)-2\left(a+bx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2]^2,x]
```

```
[Out] (-2*(a + b*x^2) + Sinh[2*(a + b*x^2)])/(8*b)
```

Maple [A] time = 0.006, size = 34, normalized size = 1.1

$$\frac{1}{2\,b}\left(\frac{\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)}{2}-\frac{bx^2}{2}-\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a)^2,x)
```

```
[Out] 1/2/b*(1/2*cosh(b*x^2+a)*sinh(b*x^2+a)-1/2*b*x^2-1/2*a)
```

Maxima [A] time = 1.13433, size = 51, normalized size = 1.65

$$-\frac{1}{4}\,x^2+\frac{e^{\left(2\,bx^2+2\,a\right)}}{16\,b}-\frac{e^{\left(-2\,bx^2-2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/16*e^(2*b*x^2 + 2*a)/b - 1/16*e^(-2*b*x^2 - 2*a)/b
```

Fricas [A] time = 1.73821, size = 68, normalized size = 2.19

$$-\frac{bx^2 - \cosh\left(bx^2 + a\right)\sinh\left(bx^2 + a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^2 - cosh(b*x^2 + a)*sinh(b*x^2 + a))/b
```

Sympy [A] time = 0.635777, size = 60, normalized size = 1.94

$$\begin{cases} \frac{x^2\sinh^2\left(a+bx^2\right)}{4} - \frac{x^2\cosh^2\left(a+bx^2\right)}{4} + \frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b} & \text{for } b \neq 0 \\ \frac{x^2\sinh^2(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x**2)**2/4 - x**2*cosh(a + b*x**2)**2/4 + sinh(a
 + b*x**2)*cosh(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*sinh(a)**2/2, True))
```

Giac [B] time = 1.14653, size = 76, normalized size = 2.45

$$-\frac{4\,bx^2 - \left(2\,e^{\left(2\,bx^2+2\,a\right)} - 1\right)e^{\left(-2\,bx^2-2\,a\right)} + 4\,a - e^{\left(2\,bx^2+2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*(4*b*x^2 - (2*e^(2*b*x^2 + 2*a) - 1)*e^(-2*b*x^2 - 2*a) + 4*a - e^(2*
b*x^2 + 2*a))/b
```

3.11 $\int \sinh^2(a+bx^2)\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{\frac{\pi}{2}}e^{-2a}\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}-\frac{x}{2}$$

```
[Out] -x/2 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)*S
qrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])
```

Rubi [A] time = 0.0458216, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5300, 5299, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{-2a}\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2,x]
```

```
[Out] -x/2 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)*S
qrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])
```

Rule 5300

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sinh^2\left(a+bx^2\right)dx &= \int \left(-\frac{1}{2}+\frac{1}{2}\cosh\left(2a+2bx^2\right)\right)dx\\ &= -\frac{x}{2}+\frac{1}{2}\int \cosh\left(2a+2bx^2\right)dx\\ &= -\frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^2}\,dx+\frac{1}{4}\int e^{2a+2bx^2}\,dx\\ &= -\frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0758382, size = 86, normalized size = 1.1

$$\frac{\sqrt{\pi}(\cosh(2a)-\sinh(2a))\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)+\sqrt{\pi}(\sinh(2a)+\cosh(2a))\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)-4\sqrt{2}\sqrt{b}x}{8\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2,x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[b]*x + Sqrt[Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2
*a]) + Sqrt[Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]))/(8*Sqrt[2]
*Sqrt[b])
```

Maple [A] time = 0.029, size = 51, normalized size = 0.7

$$-\frac{x}{2}+\frac{e^{-2\,a}\sqrt{\pi}\sqrt{2}}{16}\text{Erf}\left(x\sqrt{2}\sqrt{b}\right)\frac{1}{\sqrt{b}}+\frac{e^{2\,a}\sqrt{\pi}}{8}\text{Erf}\left(\sqrt{-2\,b}x\right)\frac{1}{\sqrt{-2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2,x)
```

```
[Out] -1/2*x+1/16*exp(-2*a)*Pi^(1/2)*2^(1/2)/b^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/8*e
xp(2*a)*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Maxima [A] time = 1.64896, size = 76, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{16\sqrt{-b}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{16\sqrt{b}}-\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) + 1/16*sqrt(
2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) - 1/2*x
```

Fricas [A] time = 1.79519, size = 225, normalized size = 2.88

$$-\frac{\sqrt{2}\sqrt{\pi}\sqrt{-b}(\cosh(2a)+\sinh(2a))\operatorname{erf}\left(\sqrt{2}\sqrt{-b}x\right)-\sqrt{2}\sqrt{\pi}\sqrt{b}(\cosh(2a)-\sinh(2a))\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)+8\,bx}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*sqrt(-b)*(cosh(2*a) + sinh(2*a))*erf(sqrt(2)*sqrt(-
b)*x) - sqrt(2)*sqrt(pi)*sqrt(b)*(cosh(2*a) - sinh(2*a))*erf(sqrt(2)*sqrt(b
)*x) + 8*b*x)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^2\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2, x)
```

Giac [A] time = 1.16841, size = 78, normalized size = 1.

$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{16\,\sqrt{-b}}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{16\,\sqrt{b}}-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) - 1/16*sqr
t(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) - 1/2*x
```

3.12 $\int \frac{\sinh^2(a+bx^2)}{x} dx$

Optimal. Leaf size=37

$$\frac{1}{4}\cosh(2a)\mathbf{Chi}\left(2bx^2\right) + \frac{1}{4}\sinh(2a)\mathbf{Shi}\left(2bx^2\right) - \frac{\log(x)}{2}$$

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2])/4 - Log[x]/2 + (Sinh[2*a]*SinhIntegral[2*
b*x^2])/4
```

Rubi [A] time = 0.0585699, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5340, 5319, 5317, 5316}

$$\frac{1}{4}\cosh(2a)\mathbf{Chi}\left(2bx^2\right) + \frac{1}{4}\sinh(2a)\mathbf{Shi}\left(2bx^2\right) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2])/4 - Log[x]/2 + (Sinh[2*a]*SinhIntegral[2*
b*x^2])/4
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5319

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2\left(a+bx^2\right)}{x}\,dx &= \int \left(-\frac{1}{2x}+\frac{\cosh\left(2a+2bx^2\right)}{2x}\right)dx\\
&= -\frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^2\right)}{x}\,dx\\
&= -\frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh\left(2bx^2\right)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh\left(2bx^2\right)}{x}\,dx\\
&= \frac{1}{4}\cosh(2a)\mathbf{Chi}\left(2bx^2\right)-\frac{\log(x)}{2}+\frac{1}{4}\sinh(2a)\mathbf{Shi}\left(2bx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0202852, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(\cosh(2a)\mathbf{Chi}\left(2bx^2\right)+\sinh(2a)\mathbf{Shi}\left(2bx^2\right)-2\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2] - 2*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^
2])/4
```

Maple [A] time = 0.033, size = 34, normalized size = 0.9

$$-\frac{\ln(x)}{2}-\frac{e^{-2a}\mathrm{Ei}\left(1,2bx^2\right)}{8}-\frac{e^{2a}\mathrm{Ei}\left(1,-2bx^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2/x,x)
```

```
[Out] -1/2*ln(x)-1/8*exp(-2*a)*Ei(1,2*b*x^2)-1/8*exp(2*a)*Ei(1,-2*b*x^2)
```

Maxima [A] time = 1.35194, size = 42, normalized size = 1.14

$$\frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} - \frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) - 1/2*log(x)
```

Fricas [A] time = 1.86007, size = 138, normalized size = 3.73

$$\frac{1}{8}\left(\mathrm{Ei}\left(2\,bx^2\right) + \mathrm{Ei}\left(-2\,bx^2\right)\right)\cosh\left(2\,a\right) + \frac{1}{8}\left(\mathrm{Ei}\left(2\,bx^2\right) - \mathrm{Ei}\left(-2\,bx^2\right)\right)\sinh\left(2\,a\right) - \frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(2*b*x^2) + Ei(-2*b*x^2))*cosh(2*a) + 1/8*(Ei(2*b*x^2) - Ei(-2*b*x^2
))*sinh(2*a) - 1/2*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2\left(a + bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2/x, x)
```

Giac [A] time = 1.1852, size = 47, normalized size = 1.27

$$\frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} - \frac{1}{4}\,\log\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) - 1/4*log(b*x^2)
```

3.13 $\int \frac{\sinh^2\left(a+bx^2\right)}{x^2} \, dx$

Optimal. Leaf size=88

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{-2a}\sqrt{b}\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{2a}\sqrt{b}\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)-\frac{\sinh^2\left(a+bx^2\right)}{x}$$

```
[Out] -(Sqrt[b]*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(2*E^(2*a)) + (Sqrt[b]*E^(2*a)
*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/2 - Sinh[a + b*x^2]^2/x
```

Rubi [A] time = 0.0676429, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 5617, 5314, 5298, 2204, 2205}

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{-2a}\sqrt{b}\text{Erf}\left(\sqrt{2}\sqrt{b}x\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{2a}\sqrt{b}\text{Erfi}\left(\sqrt{2}\sqrt{b}x\right)-\frac{\sinh^2\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2/x^2,x]
```

```
[Out] -(Sqrt[b]*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(2*E^(2*a)) + (Sqrt[b]*E^(2*a)
*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/2 - Sinh[a + b*x^2]^2/x
```

Rule 5330

```
Int[(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> -Simp[Sinh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[(b*n*p)/(n - 1), Int[Sinh[a + b
*x^n]^(p - 1)*Cosh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 5314

```
Int[((a_.) + (b_.)*Sinh[u_])^(p_.), x_Symbol] :> Int[(a + b*Sinh[ExpandToSu
m[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatc
```

```
hQ[u, x]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sinh^2\left(a+bx^2\right)}{x} + (4b)\int \cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)\,dx \\
&= -\frac{\sinh^2\left(a+bx^2\right)}{x} + (2b)\int \sinh\left(2\left(a+bx^2\right)\right)\,dx \\
&= -\frac{\sinh^2\left(a+bx^2\right)}{x} + (2b)\int \sinh\left(2a+2bx^2\right)\,dx \\
&= -\frac{\sinh^2\left(a+bx^2\right)}{x} - b\int e^{-2a-2bx^2}\,dx + b\int e^{2a+2bx^2}\,dx \\
&= -\frac{1}{2}\sqrt{b}e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right) + \frac{1}{2}\sqrt{b}e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{b}x\right) - \frac{\sinh^2\left(a+bx^2\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.225957, size = 94, normalized size = 1.07

$$\frac{\sqrt{2\pi}\sqrt{b}x(\sinh(2a)-\cosh(2a))\mathrm{Erf}\left(\sqrt{2}\sqrt{b}x\right)+\sqrt{2\pi}\sqrt{b}x(\sinh(2a)+\cosh(2a))\mathrm{Erfi}\left(\sqrt{2}\sqrt{b}x\right)-4\sinh^2\left(a+bx^2\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[2*Pi]*x*Erf[Sqrt[2]*Sqrt[b]*x]*(-Cosh[2*a] + Sinh[2*a]) + Sqr
t[b]*Sqrt[2*Pi]*x*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) - 4*Sinh[
a + b*x^2]^2)/(4*x)
```

Maple [A] time = 0.038, size = 86, normalized size = 1.

$$\frac{1}{2\,x}-\frac{e^{-2\,a}e^{-2\,bx^2}}{4\,x}-\frac{e^{-2\,a}\sqrt{\pi}\sqrt{2}}{4}\sqrt{b}\mathrm{Erf}\left(x\sqrt{2}\sqrt{b}\right)-\frac{e^{2\,a}e^{2\,bx^2}}{4\,x}+\frac{e^{2\,a}b\sqrt{\pi}}{2}\mathrm{Erf}\left(\sqrt{-2\,b}x\right)\frac{1}{\sqrt{-2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2/x^2,x)
```

```
[Out] 1/2/x-1/4*exp(-2*a)/x*exp(-2*b*x^2)-1/4*exp(-2*a)*b^(1/2)*Pi^(1/2)*2^(1/2)*
erf(x*2^(1/2)*b^(1/2))-1/4*exp(2*a)/x*exp(2*b*x^2)+1/2*exp(2*a)*b*Pi^(1/2)/
(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Maxima [A] time = 1.17014, size = 82, normalized size = 0.93

$$-\frac{\sqrt{2}\sqrt{bx^2}e^{(-2\,a)}\Gamma\left(-\frac{1}{2},2\,bx^2\right)}{8\,x}-\frac{\sqrt{2}\sqrt{-bx^2}e^{(2\,a)}\Gamma\left(-\frac{1}{2},-2\,bx^2\right)}{8\,x}+\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(b*x^2)*e^(-2*a)*gamma(-1/2, 2*b*x^2)/x - 1/8*sqrt(2)*sqrt
(-b*x^2)*e^(2*a)*gamma(-1/2, -2*b*x^2)/x + 1/2/x
```

Fricas [B] time = 1.86221, size = 1058, normalized size = 12.02

$$-\frac{\cosh\left(bx^2+a\right)^4+4\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3+\sinh\left(bx^2+a\right)^4+\sqrt{2}\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)^2\cosh\left(2\,a\right)+x\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^4 + 4*cosh(b*x^2 + a)*sinh(b*x^2 + a)^3 + sinh(b*x^2
+ a)^4 + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a) + x*cosh(b*x^2 + a
)^2*sinh(2*a) + (x*cosh(2*a) + x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cosh(b
*x^2 + a)*cosh(2*a) + x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b
)*erf(sqrt(2)*sqrt(-b)*x) + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a)
 - x*cosh(b*x^2 + a)^2*sinh(2*a) + (x*cosh(2*a) - x*sinh(2*a))*sinh(b*x^2 +
 a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) - x*cosh(b*x^2 + a)*sinh(2*a))*sinh(
b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) + 2*(3*cosh(b*x^2 + a)^2 - 1)*si
nh(b*x^2 + a)^2 - 2*cosh(b*x^2 + a)^2 + 4*(cosh(b*x^2 + a)^3 - cosh(b*x^2 +
 a))*sinh(b*x^2 + a) + 1)/(x*cosh(b*x^2 + a)^2 + 2*x*cosh(b*x^2 + a)*sinh(b
*x^2 + a) + x*sinh(b*x^2 + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^2+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^2 + a)^2/x^2, x)
```

3.14 $\int \frac{\sinh^2(a+bx^2)}{x^3} dx$

Optimal. Leaf size=57

$$\frac{1}{2}b\sinh(2a)\mathrm{Chi}\left(2bx^2\right)+\frac{1}{2}b\cosh(2a)\mathrm{Shi}\left(2bx^2\right)-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{4x^2}$$

```
[Out] 1/(4*x^2) - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2*a
])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2
```

Rubi [A] time = 0.121057, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5340, 5321, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\sinh(2a)\mathrm{Chi}\left(2bx^2\right)+\frac{1}{2}b\cosh(2a)\mathrm{Shi}\left(2bx^2\right)-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2/x^3,x]
```

```
[Out] 1/(4*x^2) - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2*a
])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5321

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2\left(a+bx^2\right)}{x^3}\,dx &= \int \left(-\frac{1}{2x^3}+\frac{\cosh\left(2a+2bx^2\right)}{2x^3}\right)dx \\
&= \frac{1}{4x^2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^2\right)}{x^3}\,dx \\
&= \frac{1}{4x^2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^2\right) \\
&= \frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{\sinh(2a+2bx)}{x}\,dx,x,x^2\right) \\
&= \frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}(b\cosh(2a))\operatorname{Subst}\left(\int \frac{\sinh(2bx)}{x}\,dx,x,x^2\right)+\frac{1}{2}(b\sinh(2a))\,\text{Subs} \\
&= \frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}b\,\mathbf{Chi}\left(2bx^2\right)\sinh(2a)+\frac{1}{2}b\cosh(2a)\mathbf{Shi}\left(2bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0906949, size = 46, normalized size = 0.81

$$\frac{1}{2}\left(b\sinh(2a)\text{Chi}\left(2bx^2\right)+b\cosh(2a)\text{Shi}\left(2bx^2\right)-\frac{\sinh^2\left(a+bx^2\right)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2/x^3,x]
```

```
[Out] (b*CoshIntegral[2*b*x^2]*Sinh[2*a] - Sinh[a + b*x^2]^2/x^2 + b*Cosh[2*a]*Si
nhIntegral[2*b*x^2])/2
```

Maple [A] time = 0.034, size = 69, normalized size = 1.2

$$\frac{1}{4\,x^2}-\frac{e^{-2\,a}e^{-2\,bx^2}}{8\,x^2}+\frac{e^{-2\,a}b\text{Ei}\left(1,2\,bx^2\right)}{4}-\frac{e^{2\,a}e^{2\,bx^2}}{8\,x^2}-\frac{e^{2\,a}b\text{Ei}\left(1,-2\,bx^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2/x^3,x)
```

```
[Out] 1/4/x^2-1/8*exp(-2*a)/x^2*exp(-2*b*x^2)+1/4*exp(-2*a)*b*Ei(1,2*b*x^2)-1/8*e
xp(2*a)/x^2*exp(2*b*x^2)-1/4*exp(2*a)*b*Ei(1,-2*b*x^2)
```

Maxima [A] time = 1.38652, size = 49, normalized size = 0.86

$$-\frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx^2\right)+\frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx^2\right)+\frac{1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^(-2*a)*gamma(-1, 2*b*x^2) + 1/4*b*e^(2*a)*gamma(-1, -2*b*x^2) + 1/
4/x^2
```

Fricas [A] time = 1.75917, size = 216, normalized size = 3.79

$$-\frac{\cosh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}\left(2\,bx^2\right)-bx^2\mathrm{Ei}\left(-2\,bx^2\right)\right)\cosh\left(2\,a\right)+\sinh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}\left(2\,bx^2\right)+bx^2\mathrm{Ei}\left(-2\,bx^2\right)\right)\sin}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) - b*x^2*Ei(-2*b*x^2))*cosh(2*a
) + sinh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) + b*x^2*Ei(-2*b*x^2))*sinh(2*a)
- 1)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^2\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2/x**3, x)
```

Giac [B] time = 1.13785, size = 170, normalized size = 2.98

$$\frac{2\left(bx^2+a\right)b^2\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)}-2\,ab^2\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)}-2\left(bx^2+a\right)b^2\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)}+2\,ab^2\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)}-b^2e^{\left(2\,bx^2+2\right.}}{8\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)*b^2*Ei(2*b*x^2)*e^(2*a) - 2*a*b^2*Ei(2*b*x^2)*e^(2*a) -
2*(b*x^2 + a)*b^2*Ei(-2*b*x^2)*e^(-2*a) + 2*a*b^2*Ei(-2*b*x^2)*e^(-2*a) - b
^2*e^(2*b*x^2 + 2*a) - b^2*e^(-2*b*x^2 - 2*a) + 2*b^2)/(b^2*x^2)
```

3.15 $\int x^3 \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=79

$$-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}+\frac{\sinh\left(a+bx^2\right)}{3b^2}-\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{x^2\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{6b}$$

```
[Out] -(x^2*Cosh[a + b*x^2])/(3*b) + Sinh[a + b*x^2]/(3*b^2) + (x^2*Cosh[a + b*x^
2]*Sinh[a + b*x^2]^2)/(6*b) - Sinh[a + b*x^2]^3/(18*b^2)
```

Rubi [A] time = 0.0831019, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5320, 3310, 3296, 2637}

$$-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}+\frac{\sinh\left(a+bx^2\right)}{3b^2}-\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{x^2\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^2]^3,x]
```

```
[Out] -(x^2*Cosh[a + b*x^2])/(3*b) + Sinh[a + b*x^2]/(3*b^2) + (x^2*Cosh[a + b*x^
2]*Sinh[a + b*x^2]^2)/(6*b) - Sinh[a + b*x^2]^3/(18*b^2)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3\sinh^3\left(a+bx^2\right)dx &= \frac{1}{2}\,\text{Subst}\left(\int x\sinh^3(a+bx)\,dx,x,x^2\right)\\ &= \frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b}-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}-\frac{1}{3}\,\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,x^2\right)\\ &= -\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b}-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}+\frac{\text{Subst}\left(\int\cosh(\right.}{3\right.}\\ &= -\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{\sinh\left(a+bx^2\right)}{3b^2}+\frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b}-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}\end{aligned}$$

Mathematica [A] time = 0.128452, size = 58, normalized size = 0.73

$$-\frac{-27\sinh\left(a+bx^2\right)+\sinh\left(3\left(a+bx^2\right)\right)+27bx^2\cosh\left(a+bx^2\right)-3bx^2\cosh\left(3\left(a+bx^2\right)\right)}{72b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^2]^3,x]
```

```
[Out] -(27*b*x^2*Cosh[a + b*x^2] - 3*b*x^2*Cosh[3*(a + b*x^2)] - 27*Sinh[a + b*x^
2] + Sinh[3*(a + b*x^2)])/(72*b^2)
```

Maple [A] time = 0.041, size = 93, normalized size = 1.2

$$\frac{\left(3\,bx^2-1\right)e^{3\,bx^2+3\,a}}{144\,b^2}-\frac{\left(3\,bx^2-3\right)e^{bx^2+a}}{16\,b^2}-\frac{\left(3\,bx^2+3\right)e^{-bx^2-a}}{16\,b^2}+\frac{\left(3\,bx^2+1\right)e^{-3\,bx^2-3\,a}}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^2+a)^3,x)
```

```
[Out] 1/144*(3*b*x^2-1)/b^2*exp(3*b*x^2+3*a)-3/16*(b*x^2-1)/b^2*exp(b*x^2+a)-3/16
*(b*x^2+1)/b^2*exp(-b*x^2-a)+1/144*(3*b*x^2+1)/b^2*exp(-3*b*x^2-3*a)
```

Maxima [A] time = 1.09418, size = 135, normalized size = 1.71

$$\frac{\left(3\,bx^2e^{(3\,a)} - e^{(3\,a)}\right)e^{\left(3\,bx^2\right)}}{144\,b^2} - \frac{3\left(bx^2e^a - e^a\right)e^{\left(bx^2\right)}}{16\,b^2} - \frac{3\left(bx^2+1\right)e^{\left(-bx^2-a\right)}}{16\,b^2} + \frac{\left(3\,bx^2+1\right)e^{\left(-3\,bx^2-3\,a\right)}}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/144*(3*b*x^2*e^(3*a) - e^(3*a))*e^(3*b*x^2)/b^2 - 3/16*(b*x^2*e^a - e^a)*
e^(b*x^2)/b^2 - 3/16*(b*x^2 + 1)*e^(-b*x^2 - a)/b^2 + 1/144*(3*b*x^2 + 1)*e
^(-3*b*x^2 - 3*a)/b^2
```

Fricas [A] time = 1.77864, size = 234, normalized size = 2.96

$$\frac{3\,bx^2\cosh\left(bx^2+a\right)^3 + 9\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2 - 27\,bx^2\cosh\left(bx^2+a\right) - \sinh\left(bx^2+a\right)^3 - 3\left(\cosh\left(bx^2+\right.\right.}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(3*b*x^2*cosh(b*x^2 + a)^3 + 9*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2
 - 27*b*x^2*cosh(b*x^2 + a) - sinh(b*x^2 + a)^3 - 3*(cosh(b*x^2 + a)^2 - 9)
*sinh(b*x^2 + a))/b^2
```

Sympy [A] time = 4.68364, size = 92, normalized size = 1.16

$$\begin{cases} \frac{x^2\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b} - \frac{x^2\cosh^3\left(a+bx^2\right)}{3b} - \frac{7\sinh^3\left(a+bx^2\right)}{18b^2} + \frac{\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b^2} & \text{for } b \neq 0 \\ \frac{x^4\sinh^3(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**2+a)**3,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x**2)**2*cosh(a + b*x**2)/(2*b) - x**2*cosh(a +
b*x**2)**3/(3*b) - 7*sinh(a + b*x**2)**3/(18*b**2) + sinh(a + b*x**2)*cosh(
a + b*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sinh(a)**3/4, True))
```

Giac [B] time = 1.19188, size = 247, normalized size = 3.13

$$\frac{3\left(bx^2+a\right)e^{\left(3\,bx^2+3\,a\right)} - 3\,ae^{\left(3\,bx^2+3\,a\right)} - 27\left(bx^2+a\right)e^{\left(bx^2+a\right)} + 27\,ae^{\left(bx^2+a\right)} - 27\left(bx^2+a\right)e^{\left(-bx^2-a\right)} + 27\,ae^{\left(-bx^2-a\right)} + 3\left(bx^2+\right.}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/144*(3*(b*x^2 + a)*e^(3*b*x^2 + 3*a) - 3*a*e^(3*b*x^2 + 3*a) - 27*(b*x^2
+ a)*e^(b*x^2 + a) + 27*a*e^(b*x^2 + a) - 27*(b*x^2 + a)*e^(-b*x^2 - a) + 2
7*a*e^(-b*x^2 - a) + 3*(b*x^2 + a)*e^(-3*b*x^2 - 3*a) - 3*a*e^(-3*b*x^2 - 3
*a) - e^(3*b*x^2 + 3*a) + 27*e^(b*x^2 + a) - 27*e^(-b*x^2 - a) + e^(-3*b*x^
2 - 3*a))/b^2
```

3.16 $\int x^2 \sinh^3\left(a+bx^2\right)dx$

Optimal. Leaf size=160

$$\frac{3\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\text{Erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3\sqrt{\pi}e^{a}\text{Erfi}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a}\text{Erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}-\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(\right.}{}$$

```
[Out] (-3*x*Cosh[a + b*x^2])/(8*b) + (x*Cosh[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi]
*Erf[Sqrt[b]*x])/(32*b^(3/2)*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[b]*x])/(96
*b^(3/2)*E^(3*a)) + (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(32*b^(3/2)) - (E^(3*a
)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2))
```

Rubi [A] time = 0.138398, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5340, 5324, 5299, 2204, 2205}

$$\frac{3\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\text{Erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3\sqrt{\pi}e^{a}\text{Erfi}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a}\text{Erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}-\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-3*x*Cosh[a + b*x^2])/(8*b) + (x*Cosh[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi]
*Erf[Sqrt[b]*x])/(32*b^(3/2)*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[b]*x])/(96
*b^(3/2)*E^(3*a)) + (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(32*b^(3/2)) - (E^(3*a
)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2))
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh^3\left(a+bx^2\right)dx &= \int\left(-\frac{3}{4}x^2\sinh\left(a+bx^2\right)+\frac{1}{4}x^2\sinh\left(3a+3bx^2\right)\right)dx \\
&= \frac{1}{4}\int x^2\sinh\left(3a+3bx^2\right)dx-\frac{3}{4}\int x^2\sinh\left(a+bx^2\right)dx \\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}-\frac{\int\cosh\left(3a+3bx^2\right)dx}{24b}+\frac{3\int\cosh\left(a+bx^2\right)dx}{8b} \\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}-\frac{\int e^{-3a-3bx^2}dx}{48b}-\frac{\int e^{3a+3bx^2}dx}{48b}+\frac{3\int e^{-a-bx^2}dx}{16b}+ \\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}+\frac{3e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3e^{a}}{}
\end{aligned}
$$

Mathematica [A] time = 0.305033, size = 184, normalized size = 1.15

$$
\frac{27\sqrt{\pi}(\cosh(a)-\sinh(a))\text{Erf}\left(\sqrt{b}x\right)+\sqrt{3\pi}(\sinh(3a)-\cosh(3a))\text{Erf}\left(\sqrt{3}\sqrt{b}x\right)+27\sqrt{\pi}\sinh(a)\text{Erfi}\left(\sqrt{b}x\right)-\sqrt{3\pi}\sinh(3}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-108*Sqrt[b]*x*Cosh[a + b*x^2] + 12*Sqrt[b]*x*Cosh[3*(a + b*x^2)] + 27*Sqr
t[Pi]*Cosh[a]*Erfi[Sqrt[b]*x] - Sqrt[3*Pi]*Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x
] + 27*Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + 27*Sqrt[Pi]*Erfi[Sqrt[
b]*x]*Sinh[a] - Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Sqrt[3*Pi]*E
rf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] + Sinh[3*a]))/(288*b^(3/2))
```

Maple [A] time = 0.07, size = 157, normalized size = 1.

$$\frac{e^{-3a}xe^{-3bx^2}}{48b}-\frac{e^{-3a}\sqrt{\pi}\sqrt{3}}{288}\mathrm{Erf}\left(x\sqrt{3}\sqrt{b}\right)b^{-\frac{3}{2}}-\frac{3e^{-a}xe^{-bx^2}}{16b}+\frac{3e^{-a}\sqrt{\pi}}{32}\mathrm{Erf}\left(x\sqrt{b}\right)b^{-\frac{3}{2}}+\frac{e^{3a}xe^{3bx^2}}{48b}-\frac{e^{3a}\sqrt{\pi}}{96b}\mathrm{Erf}\left(\sqrt{-3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(b*x^2+a)^3,x)
```

```
[Out] 1/48*exp(-3*a)/b*x*exp(-3*b*x^2)-1/288*exp(-3*a)/b^(3/2)*Pi^(1/2)*3^(1/2)*e
rf(x*3^(1/2)*b^(1/2))-3/16*exp(-a)/b*x*exp(-b*x^2)+3/32*exp(-a)/b^(3/2)*Pi^
(1/2)*erf(x*b^(1/2))+1/48*exp(3*a)/b*x*exp(3*b*x^2)-1/96*exp(3*a)/b*Pi^(1/2
)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)-3/16*exp(a)*exp(b*x^2)*x/b+3/32*exp(a)/b
*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [A] time = 1.82847, size = 219, normalized size = 1.37

$$-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{288\sqrt{-b}b}-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{\left(3bx^2+3a\right)}}{48b}-\frac{3xe^{\left(bx^2+a\right)}}{16b}-\frac{3xe^{\left(-bx^2-a\right)}}{16b}+\frac{xe^{\left(-3bx^2-3a\right)}}{48b}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) - 1/28
8*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b*
x^2 + 3*a)/b - 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b + 1/48*x*e^
(-3*b*x^2 - 3*a)/b + 3/32*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(3/2) + 3/32*sqr
t(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Fricas [B] time = 1.89479, size = 2429, normalized size = 15.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/288*(6*b*x*cosh(b*x^2 + a)^6 + 36*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 +
 6*b*x*sinh(b*x^2 + a)^6 - 54*b*x*cosh(b*x^2 + a)^4 + 18*(5*b*x*cosh(b*x^2
+ a)^2 - 3*b*x)*sinh(b*x^2 + a)^4 - 54*b*x*cosh(b*x^2 + a)^2 + 24*(5*b*x*co
sh(b*x^2 + a)^3 - 9*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)*sqrt(p
i)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) + sinh(3*a))*sinh(b*x^2 + a)^3
 + cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) + cosh(b*x^2
+ a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) + cosh(b
*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(3)*sqrt(-b)*x) -
sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) - sinh(3*a))*sin
h(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a)
 - cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh
(3*a) - cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*s
qrt(b)*x) - 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) + sinh(a))*si
nh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) +
cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) +
 cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 27*
sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a)^3
 - cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) - cosh(b*x^2 + a)
*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) - cosh(b*x^2 + a
)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 18*(5*b*x*cosh(b*x^2
 + a)^4 - 18*b*x*cosh(b*x^2 + a)^2 - 3*b*x)*sinh(b*x^2 + a)^2 + 6*b*x + 36*
(b*x*cosh(b*x^2 + a)^5 - 6*b*x*cosh(b*x^2 + a)^3 - 3*b*x*cosh(b*x^2 + a))*s
inh(b*x^2 + a))/(b^2*cosh(b*x^2 + a)^3 + 3*b^2*cosh(b*x^2 + a)^2*sinh(b*x^2
 + a) + 3*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 + b^2*sinh(b*x^2 + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh^3\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(b*x**2+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2)**3, x)
```

Giac [A] time = 1.23862, size = 224, normalized size = 1.4

$$\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{-b}x\right)e^{(3\,a)}}{288\,\sqrt{-b}b}+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{b}x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{\left(3\,bx^2+3\,a\right)}}{48\,b}-\frac{3\,xe^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,xe^{\left(-bx^2-a\right)}}{16\,b}+\frac{xe^{\left(-3\,bx^2-3\,a\right)}}{48\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/288*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) + 1/28
8*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b
*x^2 + 3*a)/b - 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b + 1/48*x*e
^(-3*b*x^2 - 3*a)/b - 3/32*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/b^(3/2) - 3/32*s
qrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

3.17 $\int x \sinh^3(a + bx^2) \, dx$

Optimal. Leaf size=33

$$\frac{\cosh^3(a+bx^2)}{6b} - \frac{\cosh(a+bx^2)}{2b}$$

```
[Out] -Cosh[a + b*x^2]/(2*b) + Cosh[a + b*x^2]^3/(6*b)
```

Rubi [A] time = 0.0330922, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2633}

$$\frac{\cosh^3(a+bx^2)}{6b} - \frac{\cosh(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2]^3,x]
```

```
[Out] -Cosh[a + b*x^2]/(2*b) + Cosh[a + b*x^2]^3/(6*b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x\sinh^3\left(a+bx^2\right)\,dx=\frac{1}{2}\,\text{Subst}\left(\int\sinh^3(a+bx)\,dx,x,x^2\right)$$
$$=-\frac{\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh\left(a+bx^2\right)\right)}{2b}$$
$$=-\frac{\cosh\left(a+bx^2\right)}{2b}+\frac{\cosh^3\left(a+bx^2\right)}{6b}$$

Mathematica [A] time = 0.013943, size = 33, normalized size = 1.

$$\frac{\cosh\left(3\left(a+bx^2\right)\right)}{24b}-\frac{3\cosh\left(a+bx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-3*Cosh[a + b*x^2])/(8*b) + Cosh[3*(a + b*x^2)]/(24*b)
```

Maple [A] time = 0.006, size = 28, normalized size = 0.9

$$\frac{\cosh\left(bx^2+a\right)}{2\,b}\left(-\frac{2}{3}+\frac{\left(\sinh\left(bx^2+a\right)\right)^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a)^3,x)
```

```
[Out] 1/2/b*(-2/3+1/3*sinh(b*x^2+a)^2)*cosh(b*x^2+a)
```

Maxima [B] time = 1.05527, size = 84, normalized size = 2.55

$$\frac{e^{\left(3\,bx^2+3\,a\right)}}{48\,b}-\frac{3\,e^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,e^{\left(-bx^2-a\right)}}{16\,b}+\frac{e^{\left(-3\,bx^2-3\,a\right)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*e^(3*b*x^2 + 3*a)/b - 3/16*e^(b*x^2 + a)/b - 3/16*e^(-b*x^2 - a)/b + 1
/48*e^(-3*b*x^2 - 3*a)/b
```

Fricas [A] time = 1.78307, size = 116, normalized size = 3.52

$$\frac{\cosh\left(bx^2+a\right)^3+3\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-9\cosh\left(bx^2+a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 - 9*cosh(b*x^
2 + a))/b
```

Sympy [A] time = 1.40858, size = 44, normalized size = 1.33

$$\begin{cases} \frac{\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b}-\frac{\cosh^3\left(a+bx^2\right)}{3b} & \text{for } b\neq 0 \\ \frac{x^2\sinh^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x**2)**2*cosh(a + b*x**2)/(2*b) - cosh(a + b*x**2)**3
/(3*b), Ne(b, 0)), (x**2*sinh(a)**3/2, True))
```

Giac [A] time = 1.22626, size = 76, normalized size = 2.3

$$-\frac{\left(9\,e^{\left(2\,bx^2+2\,a\right)}-1\right)e^{\left(-3\,bx^2-3\,a\right)}-e^{\left(3\,bx^2+3\,a\right)}+9\,e^{\left(bx^2+a\right)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*((9*e^(2*b*x^2 + 2*a) - 1)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2 + 3*a) + 9
*e^(b*x^2 + a))/b
```

3.18 $\int \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=125

$$\frac{3\sqrt{\pi}e^{-a}\mathrm{Erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\mathrm{Erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}-\frac{3\sqrt{\pi}e^{a}\mathrm{Erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a}\mathrm{Erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}$$

```
[Out] (3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(16*Sqrt[b]*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt
[b]*x])/(16*Sqrt[b]*E^(3*a)) - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(16*Sqrt[b]
) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b])
```

Rubi [A] time = 0.0718229, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5300, 5298, 2204, 2205}

$$\frac{3\sqrt{\pi}e^{-a}\mathrm{Erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\mathrm{Erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}-\frac{3\sqrt{\pi}e^{a}\mathrm{Erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a}\mathrm{Erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(16*Sqrt[b]*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt
[b]*x])/(16*Sqrt[b]*E^(3*a)) - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(16*Sqrt[b]
) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b])
```

Rule 5300

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sinh^3\left(a+bx^2\right)dx &= \int \left(-\frac{3}{4}\sinh\left(a+bx^2\right)+\frac{1}{4}\sinh\left(3a+3bx^2\right)\right)dx\\ &= \frac{1}{4}\int \sinh\left(3a+3bx^2\right)dx-\frac{3}{4}\int \sinh\left(a+bx^2\right)dx\\ &= -\left(\frac{1}{8}\int e^{-3a-3bx^2}\,dx\right)+\frac{1}{8}\int e^{3a+3bx^2}\,dx+\frac{3}{8}\int e^{-a-bx^2}\,dx-\frac{3}{8}\int e^{a+bx^2}\,dx\\ &= \frac{3e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}-\frac{3e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.136555, size = 136, normalized size = 1.09

$$\frac{\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}(\cosh(a)-\sinh(a))\operatorname{Erf}\left(\sqrt{b}x\right)+(\sinh(3a)-\cosh(3a))\operatorname{Erf}\left(\sqrt{3}\sqrt{b}x\right)-3\sqrt{3}\sinh(a)\operatorname{Erfi}\left(\sqrt{b}x\right)+\sinh(3a)\operatorname{Erfi}}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[a]*Erfi[Sqrt[b]*x] + Cosh[3*a]*Erfi[Sqrt[3]*Sq
rt[b]*x] + 3*Sqrt[3]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - 3*Sqrt[3]*Erfi[Sq
rt[b]*x]*Sinh[a] + Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Erf[Sqrt[3]*Sqrt[b]*
x]*(-Cosh[3*a] + Sinh[3*a])))/(16*Sqrt[b])
```

Maple [A] time = 0.036, size = 86, normalized size = 0.7

$$-\frac{e^{-3a}\sqrt{\pi}\sqrt{3}}{48}\operatorname{Erf}\left(x\sqrt{3}\sqrt{b}\right)\frac{1}{\sqrt{b}}+\frac{3\sqrt{\pi}e^{-a}}{16}\operatorname{Erf}\left(x\sqrt{b}\right)\frac{1}{\sqrt{b}}+\frac{e^{3a}\sqrt{\pi}}{16}\operatorname{Erf}\left(\sqrt{-3b}x\right)\frac{1}{\sqrt{-3b}}-\frac{3e^{a}\sqrt{\pi}}{16}\operatorname{Erf}\left(\sqrt{-b}x\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3,x)
```

```
[Out] -1/48*exp(-3*a)*Pi^(1/2)*3^(1/2)/b^(1/2)*erf(x*3^(1/2)*b^(1/2))+3/16*erf(x*
b^(1/2))*Pi^(1/2)*exp(-a)/b^(1/2)+1/16*exp(3*a)*Pi^(1/2)/(-3*b)^(1/2)*erf((
-3*b)^(1/2)*x)-3/16*exp(a)*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [A] time = 1.6408, size = 123, normalized size = 0.98

$$\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{48\sqrt{-b}} - \frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{48\sqrt{b}} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{16\sqrt{b}} - \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right)e^{a}}{16\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) - 1/48*sqrt(
3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqr
t(b)*x)*e^(-a)/sqrt(b) - 3/16*sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b)
```

Fricas [A] time = 1.79963, size = 369, normalized size = 2.95

$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-b}(\cosh(3a) + \sinh(3a))\operatorname{erf}\left(\sqrt{3}\sqrt{-b}x\right) + \sqrt{3}\sqrt{\pi}\sqrt{b}(\cosh(3a) - \sinh(3a))\operatorname{erf}\left(\sqrt{3}\sqrt{b}x\right) - 9\sqrt{\pi}\sqrt{-b}(\cosh(}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-b)*(cosh(3*a) + sinh(3*a))*erf(sqrt(3)*sqrt(-
b)*x) + sqrt(3)*sqrt(pi)*sqrt(b)*(cosh(3*a) - sinh(3*a))*erf(sqrt(3)*sqrt(b
)*x) - 9*sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - 9*sqrt(pi)
*sqrt(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^3\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3, x)
```

Giac [A] time = 1.26431, size = 128, normalized size = 1.02

$$-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{48\sqrt{-b}}+\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{48\sqrt{b}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{16\sqrt{b}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}x\right)e^{a}}{16\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) + 1/48*sqr
t(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) - 3/16*sqrt(pi)*erf(
-sqrt(b)*x)*e^(-a)/sqrt(b) + 3/16*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)
```

3.19 $\int \frac{\sinh^3(a+bx^2)}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{3}{8}\sinh(a)\mathbf{Chi}\left(bx^2\right)+\frac{1}{8}\sinh(3a)\mathbf{Chi}\left(3bx^2\right)-\frac{3}{8}\cosh(a)\mathbf{Shi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\mathbf{Shi}\left(3bx^2\right)$$

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a])/8 + (CoshIntegral[3*b*x^2]*Sinh[3*a])/8 -
(3*Cosh[a]*SinhIntegral[b*x^2])/8 + (Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rubi [A] time = 0.0939522, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5340, 5318, 5317, 5316}

$$-\frac{3}{8}\sinh(a)\mathbf{Chi}\left(bx^2\right)+\frac{1}{8}\sinh(3a)\mathbf{Chi}\left(3bx^2\right)-\frac{3}{8}\cosh(a)\mathbf{Shi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\mathbf{Shi}\left(3bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a])/8 + (CoshIntegral[3*b*x^2]*Sinh[3*a])/8 -
(3*Cosh[a]*SinhIntegral[b*x^2])/8 + (Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3\left(a+bx^2\right)}{x}\,dx &= \int \left(-\frac{3\sinh\left(a+bx^2\right)}{4x}+\frac{\sinh\left(3a+3bx^2\right)}{4x}\right)dx \\
&= \frac{1}{4}\int \frac{\sinh\left(3a+3bx^2\right)}{x}\,dx-\frac{3}{4}\int \frac{\sinh\left(a+bx^2\right)}{x}\,dx \\
&= -\left(\frac{1}{4}(3\cosh(a))\int \frac{\sinh\left(bx^2\right)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\sinh\left(3bx^2\right)}{x}\,dx-\frac{1}{4}(3\sinh(a))\int \frac{\cosh\left(bx\right.}{x} \\
&= -\frac{3}{8}\mathrm{Chi}\left(bx^2\right)\sinh(a)+\frac{1}{8}\mathrm{Chi}\left(3bx^2\right)\sinh(3a)-\frac{3}{8}\cosh(a)\mathrm{Shi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\mathrm{Shi}\left(3bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0336521, size = 49, normalized size = 0.89

$$\frac{1}{8}\left(-3\sinh(a)\mathrm{Chi}\left(bx^2\right)+\sinh(3a)\mathrm{Chi}\left(3bx^2\right)-3\cosh(a)\mathrm{Shi}\left(bx^2\right)+\cosh(3a)\mathrm{Shi}\left(3bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a] + CoshIntegral[3*b*x^2]*Sinh[3*a] - 3*Cosh[
a]*SinhIntegral[b*x^2] + Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Maple [A] time = 0.038, size = 55, normalized size = 1.

$$\frac{e^{-3a}\mathrm{Ei}\left(1,3bx^2\right)}{16}-\frac{3e^{-a}\mathrm{Ei}\left(1,bx^2\right)}{16}-\frac{e^{3a}\mathrm{Ei}\left(1,-3bx^2\right)}{16}+\frac{3e^{a}\mathrm{Ei}\left(1,-bx^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3/x,x)
```

```
[Out] 1/16*exp(-3*a)*Ei(1,3*b*x^2)-3/16*exp(-a)*Ei(1,b*x^2)-1/16*exp(3*a)*Ei(1,-3
*b*x^2)+3/16*exp(a)*Ei(1,-b*x^2)
```

Maxima [A] time = 1.32201, size = 68, normalized size = 1.24

$$\frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)}-\frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)}-\frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) - 1/16*Ei(-3*b*x^2)*e^(-3
*a) - 3/16*Ei(b*x^2)*e^a
```

Fricas [A] time = 1.64296, size = 231, normalized size = 4.2

$$\frac{1}{16}\left(\mathrm{Ei}\left(3\,bx^2\right)-\mathrm{Ei}\left(-3\,bx^2\right)\right)\cosh\left(3\,a\right)-\frac{3}{16}\left(\mathrm{Ei}\left(bx^2\right)-\mathrm{Ei}\left(-bx^2\right)\right)\cosh\left(a\right)+\frac{1}{16}\left(\mathrm{Ei}\left(3\,bx^2\right)+\mathrm{Ei}\left(-3\,bx^2\right)\right)\sinh\left(3\,a\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(3*b*x^2) - Ei(-3*b*x^2))*cosh(3*a) - 3/16*(Ei(b*x^2) - Ei(-b*x^2))
*cosh(a) + 1/16*(Ei(3*b*x^2) + Ei(-3*b*x^2))*sinh(3*a) - 3/16*(Ei(b*x^2) +
Ei(-b*x^2))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^3\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x, x)
```

Giac [A] time = 1.27941, size = 68, normalized size = 1.24

$$\frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)}-\frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)}-\frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) - 1/16*Ei(-3*b*x^2)*e^(-3
*a) - 3/16*Ei(b*x^2)*e^a
```

3.20 $\int \frac{\sinh^3\left(a+bx^2\right)}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{3}{8}\sqrt{\pi}e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{3\pi}e^{-3a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{b}x\right)-\frac{3}{8}\sqrt{\pi}e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{3\pi}e^{3a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{b}x\right)-\frac{\sinh^3(a+b}{x}$$

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*E^a) + (Sqrt[b]*Sqrt[3*Pi]*Erf[Sqrt
[3]*Sqrt[b]*x])/(8*E^(3*a)) - (3*Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/8 +
(Sqrt[b]*E^(3*a)*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x])/8 - Sinh[a + b*x^2]^3/
x
```

Rubi [A] time = 0.109961, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 5618, 5299, 2204, 2205}

$$-\frac{3}{8}\sqrt{\pi}e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{3\pi}e^{-3a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{b}x\right)-\frac{3}{8}\sqrt{\pi}e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{3\pi}e^{3a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{b}x\right)-\frac{\sinh^3(a+b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*E^a) + (Sqrt[b]*Sqrt[3*Pi]*Erf[Sqrt
[3]*Sqrt[b]*x])/(8*E^(3*a)) - (3*Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/8 +
(Sqrt[b]*E^(3*a)*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x])/8 - Sinh[a + b*x^2]^3/
x
```

Rule 5330

```
Int[(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> -Simp[Sinh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[(b*n*p)/(n - 1), Int[Sinh[a + b
*x^n]^(p - 1)*Cosh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
```

```
]))
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sinh^3\left(a+bx^2\right)}{x}+(6b)\int \cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)\,dx\\
&= -\frac{\sinh^3\left(a+bx^2\right)}{x}+(6b)\int \left(-\frac{1}{4}\cosh\left(a+bx^2\right)+\frac{1}{4}\cosh\left(3a+3bx^2\right)\right)dx\\
&= -\frac{\sinh^3\left(a+bx^2\right)}{x}-\frac{1}{2}(3b)\int \cosh\left(a+bx^2\right)dx+\frac{1}{2}(3b)\int \cosh\left(3a+3bx^2\right)dx\\
&= -\frac{\sinh^3\left(a+bx^2\right)}{x}+\frac{1}{4}(3b)\int e^{-3a-3bx^2}\,dx-\frac{1}{4}(3b)\int e^{-a-bx^2}\,dx-\frac{1}{4}(3b)\int e^{a+bx^2}\,dx+\frac{1}{4}(3b)\int\\
&= -\frac{3}{8}\sqrt{b}e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{b}e^{-3a}\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)-\frac{3}{8}\sqrt{b}e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{b}e^{3a}\sqrt{3\pi}\,\mathrm{erfi}
\end{aligned}
$$

Mathematica [A] time = 0.329041, size = 204, normalized size = 1.5

$$3\sqrt{\pi}\sqrt{b}x(\sinh(a)-\cosh(a))\mathrm{Erf}\left(\sqrt{b}x\right)+\sqrt{3\pi}\sqrt{b}x(\cosh(3a)-\sinh(3a))\mathrm{Erf}\left(\sqrt{3}\sqrt{b}x\right)-3\sqrt{\pi}\sqrt{b}x\sinh(a)\mathrm{Erfi}\left(\sqrt{b}x\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*x*Cosh[a]*Erfi[Sqrt[b]*x] + Sqrt[b]*Sqrt[3*Pi]*x*Cosh[
3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] - 3*Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*Sinh[a]
 + 3*Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Sqrt[b]*Sqrt[
3*Pi]*x*Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh[3*a]) + Sqrt[b]*Sqrt[3*Pi]
*x*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + 6*Sinh[a + b*x^2] - 2*Sinh[3*(a + b*
x^2)])/(8*x)
```

Maple [A] time = 0.052, size = 149, normalized size = 1.1

$$\frac{e^{-3a}e^{-3bx^2}}{8x}+\frac{e^{-3a}\sqrt{\pi}\sqrt{3}}{8}\sqrt{b}\mathrm{Erf}\left(x\sqrt{3}\sqrt{b}\right)-\frac{3e^{-a}e^{-bx^2}}{8x}-\frac{3e^{-a}\sqrt{\pi}}{8}\sqrt{b}\mathrm{Erf}\left(x\sqrt{b}\right)-\frac{e^{3a}e^{3bx^2}}{8x}+\frac{3e^{3a}b\sqrt{\pi}}{8}\mathrm{Erf}\left(\sqrt{-3b}x\right)\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3/x^2,x)
```

```
[Out] 1/8*exp(-3*a)/x*exp(-3*b*x^2)+1/8*exp(-3*a)*b^(1/2)*Pi^(1/2)*3^(1/2)*erf(x*
3^(1/2)*b^(1/2))-3/8*exp(-a)/x*exp(-b*x^2)-3/8*exp(-a)*b^(1/2)*Pi^(1/2)*erf
(x*b^(1/2))-1/8*exp(3*a)/x*exp(3*b*x^2)+3/8*exp(3*a)*b*Pi^(1/2)/(-3*b)^(1/2
)*erf((-3*b)^(1/2)*x)+3/8*exp(a)*exp(b*x^2)/x-3/8*exp(a)*b*Pi^(1/2)/(-b)^(1
/2)*erf((-b)^(1/2)*x)
```

Maxima [A] time = 1.37234, size = 138, normalized size = 1.01

$$\frac{\sqrt{3}\sqrt{bx^2}e^{(-3a)}\Gamma\left(-\frac{1}{2},3bx^2\right)}{16x}-\frac{\sqrt{3}\sqrt{-bx^2}e^{(3a)}\Gamma\left(-\frac{1}{2},-3bx^2\right)}{16x}-\frac{3\sqrt{bx^2}e^{(-a)}\Gamma\left(-\frac{1}{2},bx^2\right)}{16x}+\frac{3\sqrt{-bx^2}e^{a}\Gamma\left(-\frac{1}{2},-bx^2\right)}{16x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(3)*sqrt(b*x^2)*e^(-3*a)*gamma(-1/2, 3*b*x^2)/x - 1/16*sqrt(3)*sqr
t(-b*x^2)*e^(3*a)*gamma(-1/2, -3*b*x^2)/x - 3/16*sqrt(b*x^2)*e^(-a)*gamma(-
1/2, b*x^2)/x + 3/16*sqrt(-b*x^2)*e^a*gamma(-1/2, -b*x^2)/x
```

Fricas [B] time = 1.86949, size = 2395, normalized size = 17.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(b*x^2 + a)^6 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 + sinh(b*x^2
+ a)^6 + 3*(5*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)^4 - 3*cosh(b*x^2 + a)^
4 + 4*(5*cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)
*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) + x*cosh(b*x^2 + a)^3*sinh(3*a) +
(x*cosh(3*a) + x*sinh(3*a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(3
*a) + x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)
^2*cosh(3*a) + x*cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf
(sqrt(3)*sqrt(-b)*x) - sqrt(3)*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) - x*
cosh(b*x^2 + a)^3*sinh(3*a) + (x*cosh(3*a) - x*sinh(3*a))*sinh(b*x^2 + a)^3
 + 3*(x*cosh(b*x^2 + a)*cosh(3*a) - x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2
 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(3*a) - x*cosh(b*x^2 + a)^2*sinh(3*a))
*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*sqrt(b)*x) - 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) + x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) + x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) + x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) - x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) - x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) - x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 3*(5*cosh(b*x^2 + a)^4 - 6
*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^2 + 3*cosh(b*x^2 + a)^2 + 6*(cosh(b
*x^2 + a)^5 - 2*cosh(b*x^2 + a)^3 + cosh(b*x^2 + a))*sinh(b*x^2 + a) - 1)/(
x*cosh(b*x^2 + a)^3 + 3*x*cosh(b*x^2 + a)^2*sinh(b*x^2 + a) + 3*x*cosh(b*x^
2 + a)*sinh(b*x^2 + a)^2 + x*sinh(b*x^2 + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3\left(a + bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^2 + a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^2 + a)^3/x^2, x)
```

3.21 $\int \frac{\sinh^3(a+bx^2)}{x^3} dx$

Optimal. Leaf size=91

$$-\frac{3}{8}b\cosh(a)\text{Chi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\text{Chi}\left(3bx^2\right)-\frac{3}{8}b\sinh(a)\text{Shi}\left(bx^2\right)+\frac{3}{8}b\sinh(3a)\text{Shi}\left(3bx^2\right)+\frac{3\sinh\left(a+bx^2\right)}{8x^2}-$$

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^2])/8 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x^2]
)/8 + (3*Sinh[a + b*x^2])/(8*x^2) - Sinh[3*(a + b*x^2)]/(8*x^2) - (3*b*Sinh
[a]*SinhIntegral[b*x^2])/8 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rubi [A] time = 0.213875, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5340, 5320, 3297, 3303, 3298, 3301}

$$-\frac{3}{8}b\cosh(a)\text{Chi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\text{Chi}\left(3bx^2\right)-\frac{3}{8}b\sinh(a)\text{Shi}\left(bx^2\right)+\frac{3}{8}b\sinh(3a)\text{Shi}\left(3bx^2\right)+\frac{3\sinh\left(a+bx^2\right)}{8x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^2])/8 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x^2]
)/8 + (3*Sinh[a + b*x^2])/(8*x^2) - Sinh[3*(a + b*x^2)]/(8*x^2) - (3*b*Sinh
[a]*SinhIntegral[b*x^2])/8 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx = \int \left(-\frac{3\sinh\left(a+bx^2\right)}{4x^3}+\frac{\sinh\left(3a+3bx^2\right)}{4x^3}\right)dx$$

$$= \frac{1}{4}\int \frac{\sinh\left(3a+3bx^2\right)}{x^3}\,dx - \frac{3}{4}\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx$$

$$= \frac{1}{8}\operatorname{Subst}\left(\int \frac{\sinh(3a+3bx)}{x^2}\,dx, x, x^2\right) - \frac{3}{8}\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, x^2\right)$$

$$= \frac{3\sinh\left(a+bx^2\right)}{8x^2} - \frac{\sinh\left(3\left(a+bx^2\right)\right)}{8x^2} - \frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, x^2\right) + \frac{1}{8}(3b)\operatorname{Subst}$$

$$= \frac{3\sinh\left(a+bx^2\right)}{8x^2} - \frac{\sinh\left(3\left(a+bx^2\right)\right)}{8x^2} - \frac{1}{8}(3b\cosh(a))\operatorname{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, x^2\right) + \frac{1}{8}(3b\cos$$

$$= -\frac{3}{8}b\cosh(a)\mathbf{Chi}\left(bx^2\right) + \frac{3}{8}b\cosh(3a)\mathbf{Chi}\left(3bx^2\right) + \frac{3\sinh\left(a+bx^2\right)}{8x^2} - \frac{\sinh\left(3\left(a+bx^2\right)\right)}{8x^2} - \frac{3}{8}b\,s$$

Mathematica [A] time = 0.116758, size = 90, normalized size = 0.99

$$-\frac{3bx^2\cosh(a)\mathbf{Chi}\left(bx^2\right)-3bx^2\cosh(3a)\mathbf{Chi}\left(3bx^2\right)+3bx^2\sinh(a)\mathbf{Shi}\left(bx^2\right)-3bx^2\sinh(3a)\mathbf{Shi}\left(3bx^2\right)-3\sinh\left(a+b\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3/x^3,x]
```

```
[Out] -(3*b*x^2*Cosh[a]*CoshIntegral[b*x^2] - 3*b*x^2*Cosh[3*a]*CoshIntegral[3*b*
x^2] - 3*Sinh[a + b*x^2] + Sinh[3*(a + b*x^2)] + 3*b*x^2*Sinh[a]*SinhIntegr
al[b*x^2] - 3*b*x^2*Sinh[3*a]*SinhIntegral[3*b*x^2])/(8*x^2)
```

Maple [A] time = 0.041, size = 120, normalized size = 1.3

$$\frac{e^{-3a}e^{-3bx^2}}{16x^2}-\frac{3e^{-3a}b\mathrm{Ei}\left(1,3bx^2\right)}{16}-\frac{3e^{-a}e^{-bx^2}}{16x^2}+\frac{3e^{-a}b\mathrm{Ei}\left(1,bx^2\right)}{16}-\frac{e^{3a}e^{3bx^2}}{16x^2}-\frac{3e^{3a}b\mathrm{Ei}\left(1,-3bx^2\right)}{16}+\frac{3e^{a}e^{bx^2}}{16x^2}+\frac{3e^{a}b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3/x^3,x)
```

```
[Out] 1/16*exp(-3*a)/x^2*exp(-3*b*x^2)-3/16*exp(-3*a)*b*Ei(1,3*b*x^2)-3/16*exp(-a
)/x^2*exp(-b*x^2)+3/16*exp(-a)*b*Ei(1,b*x^2)-1/16*exp(3*a)/x^2*exp(3*b*x^2)
-3/16*exp(3*a)*b*Ei(1,-3*b*x^2)+3/16*exp(a)*exp(b*x^2)/x^2+3/16*exp(a)*b*Ei
(1,-b*x^2)
```

Maxima [A] time = 1.20758, size = 78, normalized size = 0.86

$$\frac{3}{16}be^{(-3a)}\Gamma\left(-1,3bx^2\right)-\frac{3}{16}be^{(-a)}\Gamma\left(-1,bx^2\right)-\frac{3}{16}be^{a}\Gamma\left(-1,-bx^2\right)+\frac{3}{16}be^{(3a)}\Gamma\left(-1,-3bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*b*e^(-3*a)*gamma(-1, 3*b*x^2) - 3/16*b*e^(-a)*gamma(-1, b*x^2) - 3/16*
b*e^a*gamma(-1, -b*x^2) + 3/16*b*e^(3*a)*gamma(-1, -3*b*x^2)
```

Fricas [A] time = 1.7403, size = 385, normalized size = 4.23

$$-\frac{2\sinh\left(bx^2+a\right)^3-3\left(bx^2\mathrm{Ei}\left(3\,bx^2\right)+bx^2\mathrm{Ei}\left(-3\,bx^2\right)\right)\cosh\left(3\,a\right)+3\left(bx^2\mathrm{Ei}\left(bx^2\right)+bx^2\mathrm{Ei}\left(-bx^2\right)\right)\cosh\left(a\right)+6\left(\cosh\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sinh(b*x^2 + a)^3 - 3*(b*x^2*Ei(3*b*x^2) + b*x^2*Ei(-3*b*x^2))*cos
h(3*a) + 3*(b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*cosh(a) + 6*(cosh(b*x^2 + a
)^2 - 1)*sinh(b*x^2 + a) - 3*(b*x^2*Ei(3*b*x^2) - b*x^2*Ei(-3*b*x^2))*sinh(
3*a) + 3*(b*x^2*Ei(b*x^2) - b*x^2*Ei(-b*x^2))*sinh(a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x**3, x)
```

Giac [B] time = 1.28424, size = 301, normalized size = 3.31

$$\frac{3\left(bx^2+a\right)b^2\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}-3\,ab^2\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}-3\left(bx^2+a\right)b^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+3\,ab^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+3\left(bx^2+a\right)b^2\mathrm{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(b*x^2 + a)*b^2*Ei(3*b*x^2)*e^(3*a) - 3*a*b^2*Ei(3*b*x^2)*e^(3*a) -
 3*(b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) + 3*a*b^2*Ei(-b*x^2)*e^(-a) + 3*(b*x^2
 + a)*b^2*Ei(-3*b*x^2)*e^(-3*a) - 3*a*b^2*Ei(-3*b*x^2)*e^(-3*a) - 3*(b*x^2
```

```
+ a)*b^2*Ei(b*x^2)*e^a + 3*a*b^2*Ei(b*x^2)*e^a - b^2*e^(3*b*x^2 + 3*a) + 3*
b^2*e^(b*x^2 + a) - 3*b^2*e^(-b*x^2 - a) + b^2*e^(-3*b*x^2 - 3*a))/(b^2*x^2
)
```

3.22 $\int x \sinh^7\left(a+bx^2\right)\,dx$

Optimal. Leaf size=67

$$\frac{\cosh^7\left(a+bx^2\right)}{14b}-\frac{3\cosh^5\left(a+bx^2\right)}{10b}+\frac{\cosh^3\left(a+bx^2\right)}{2b}-\frac{\cosh\left(a+bx^2\right)}{2b}$$

```
[Out] -Cosh[a + b*x^2]/(2*b) + Cosh[a + b*x^2]^3/(2*b) - (3*Cosh[a + b*x^2]^5)/(1
0*b) + Cosh[a + b*x^2]^7/(14*b)
```

Rubi [A] time = 0.0473215, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2633}

$$\frac{\cosh^7\left(a+bx^2\right)}{14b}-\frac{3\cosh^5\left(a+bx^2\right)}{10b}+\frac{\cosh^3\left(a+bx^2\right)}{2b}-\frac{\cosh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2]^7,x]
```

```
[Out] -Cosh[a + b*x^2]/(2*b) + Cosh[a + b*x^2]^3/(2*b) - (3*Cosh[a + b*x^2]^5)/(1
0*b) + Cosh[a + b*x^2]^7/(14*b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x\sinh^7\left(a+bx^2\right)dx = \frac{1}{2}\operatorname{Subst}\left(\int \sinh^7(a+bx)\,dx,x,x^2\right)$$
$$= -\frac{\operatorname{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,\cosh\left(a+bx^2\right)\right)}{2b}$$
$$= -\frac{\cosh\left(a+bx^2\right)}{2b}+\frac{\cosh^3\left(a+bx^2\right)}{2b}-\frac{3\cosh^5\left(a+bx^2\right)}{10b}+\frac{\cosh^7\left(a+bx^2\right)}{14b}$$

Mathematica [A] time = 0.0248622, size = 67, normalized size = 1.

$$-\frac{35\cosh\left(a+bx^2\right)}{128b}+\frac{7\cosh\left(3\left(a+bx^2\right)\right)}{128b}-\frac{7\cosh\left(5\left(a+bx^2\right)\right)}{640b}+\frac{\cosh\left(7\left(a+bx^2\right)\right)}{896b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2]^7,x]
```

```
[Out] (-35*Cosh[a + b*x^2])/(128*b) + (7*Cosh[3*(a + b*x^2)])/(128*b) - (7*Cosh[5
*(a + b*x^2)])/(640*b) + Cosh[7*(a + b*x^2)]/(896*b)
```

Maple [A] time = 0.045, size = 52, normalized size = 0.8

$$\frac{\cosh\left(bx^2+a\right)}{2b}\left(-\frac{16}{35}+\frac{\left(\sinh\left(bx^2+a\right)\right)^6}{7}-\frac{6\left(\sinh\left(bx^2+a\right)\right)^4}{35}+\frac{8\left(\sinh\left(bx^2+a\right)\right)^2}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a)^7,x)
```

```
[Out] 1/2/b*(-16/35+1/7*sinh(b*x^2+a)^6-6/35*sinh(b*x^2+a)^4+8/35*sinh(b*x^2+a)^2
)*cosh(b*x^2+a)
```

Maxima [B] time = 1.02966, size = 170, normalized size = 2.54

$$\frac{e^{\left(7\,bx^2+7\,a\right)}}{1792\,b}-\frac{7\,e^{\left(5\,bx^2+5\,a\right)}}{1280\,b}+\frac{7\,e^{\left(3\,bx^2+3\,a\right)}}{256\,b}-\frac{35\,e^{\left(bx^2+a\right)}}{256\,b}-\frac{35\,e^{\left(-bx^2-a\right)}}{256\,b}+\frac{7\,e^{\left(-3\,bx^2-3\,a\right)}}{256\,b}-\frac{7\,e^{\left(-5\,bx^2-5\,a\right)}}{1280\,b}+\frac{e^{\left(-7\,bx^2-7\,a\right)}}{1792\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/1792*e^(7*b*x^2 + 7*a)/b - 7/1280*e^(5*b*x^2 + 5*a)/b + 7/256*e^(3*b*x^2
+ 3*a)/b - 35/256*e^(b*x^2 + a)/b - 35/256*e^(-b*x^2 - a)/b + 7/256*e^(-3*b
*x^2 - 3*a)/b - 7/1280*e^(-5*b*x^2 - 5*a)/b + 1/1792*e^(-7*b*x^2 - 7*a)/b
```

Fricas [B] time = 1.68794, size = 398, normalized size = 5.94

$$5\cosh\left(bx^2+a\right)^7+35\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^6-49\cosh\left(bx^2+a\right)^5+35\left(5\cosh\left(bx^2+a\right)^3-7\cosh\left(bx^2+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/4480*(5*cosh(b*x^2 + a)^7 + 35*cosh(b*x^2 + a)*sinh(b*x^2 + a)^6 - 49*cos
h(b*x^2 + a)^5 + 35*(5*cosh(b*x^2 + a)^3 - 7*cosh(b*x^2 + a))*sinh(b*x^2 +
a)^4 + 245*cosh(b*x^2 + a)^3 + 35*(3*cosh(b*x^2 + a)^5 - 14*cosh(b*x^2 + a)
^3 + 21*cosh(b*x^2 + a))*sinh(b*x^2 + a)^2 - 1225*cosh(b*x^2 + a))/b
```

Sympy [A] time = 13.3207, size = 94, normalized size = 1.4

$$\begin{cases} \frac{\sinh^{6}\left(a+bx^{2}\right)\cosh\left(a+bx^{2}\right)}{2b}-\frac{\sinh^{4}\left(a+bx^{2}\right)\cosh^{3}\left(a+bx^{2}\right)}{b}+\frac{4\sinh^{2}\left(a+bx^{2}\right)\cosh^{5}\left(a+bx^{2}\right)}{5b}-\frac{8\cosh^{7}\left(a+bx^{2}\right)}{35b} & \text{for}\: b\neq 0 \\ \frac{x^{2}\sinh^{7}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a)**7,x)
```

```
[Out] Piecewise((sinh(a + b*x**2)**6*cosh(a + b*x**2)/(2*b) - sinh(a + b*x**2)**4
*cosh(a + b*x**2)**3/b + 4*sinh(a + b*x**2)**2*cosh(a + b*x**2)**5/(5*b) -
8*cosh(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sinh(a)**7/2, True))
```

Giac [A] time = 1.3114, size = 146, normalized size = 2.18

$$-\frac{\left(1225\,e^{\left(6\,bx^{2}+6\,a\right)}-245\,e^{\left(4\,bx^{2}+4\,a\right)}+49\,e^{\left(2\,bx^{2}+2\,a\right)}-5\right)e^{\left(-7\,bx^{2}-7\,a\right)}-5\,e^{\left(7\,bx^{2}+7\,a\right)}+49\,e^{\left(5\,bx^{2}+5\,a\right)}-245\,e^{\left(3\,bx^{2}+3\,a\right)}+1225\,e}{8960\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/8960*((1225*e^(6*b*x^2 + 6*a) - 245*e^(4*b*x^2 + 4*a) + 49*e^(2*b*x^2 +
2*a) - 5)*e^(-7*b*x^2 - 7*a) - 5*e^(7*b*x^2 + 7*a) + 49*e^(5*b*x^2 + 5*a) -
 245*e^(3*b*x^2 + 3*a) + 1225*e^(b*x^2 + a))/b
```

3.23 $\int (ex)^m \sinh^p\left(a+bx^2\right) dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((ex)^m \sinh^p\left(a+bx^2\right),x\right)$$

```
[Out] Unintegrable[(e*x)^m*Sinh[a + b*x^2]^p, x]
```

Rubi [A] time = 0.0196521, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^p,x]
```

```
[Out] Defer[Int][(e*x)^m*Sinh[a + b*x^2]^p, x]
```

Rubi steps

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx = \int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Mathematica [A] time = 2.3787, size = 0, normalized size = 0.

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^p,x]
```

```
[Out] Integrate[(e*x)^m*Sinh[a + b*x^2]^p, x]
```

Maple [A] time = 0.039, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh\left(bx^2+a\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a)^p,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \sinh\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^2 + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**p,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^p, x)
```

3.24 $\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=214

$$-\frac{e^{3a}3^{-\frac{m}{2}-\frac{1}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-3bx^2\right)}{16e}+\frac{3e^a\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)}{16e}-\frac{3e^{-a}\left(bx^2\right.}{}$$

```
[Out] -(3^(-1/2 - m/2)*E^(3*a)*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m
)/2, -3*b*x^2])/(16*e) + (3*E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma
[(1 + m)/2, -(b*x^2)])/(16*e) - (3*(e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma
[(1 + m)/2, b*x^2])/(16*e*E^a) + (3^(-1/2 - m/2)*(e*x)^(1 + m)*(b*x^2)^((-1
 - m)/2)*Gamma[(1 + m)/2, 3*b*x^2])/(16*e*E^(3*a))
```

Rubi [A] time = 0.201293, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5340, 5328, 2218}

$$-\frac{e^{3a}3^{-\frac{m}{2}-\frac{1}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-3bx^2\right)}{16e}+\frac{3e^a\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)}{16e}-\frac{3e^{-a}\left(bx^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^3,x]
```

```
[Out] -(3^(-1/2 - m/2)*E^(3*a)*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m
)/2, -3*b*x^2])/(16*e) + (3*E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma
[(1 + m)/2, -(b*x^2)])/(16*e) - (3*(e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma
[(1 + m)/2, b*x^2])/(16*e*E^a) + (3^(-1/2 - m/2)*(e*x)^(1 + m)*(b*x^2)^((-1
 - m)/2)*Gamma[(1 + m)/2, 3*b*x^2])/(16*e*E^(3*a))
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
```

```
x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx &= \int \left(-\frac{3}{4}(ex)^m \sinh\left(a+bx^2\right)+\frac{1}{4}(ex)^m \sinh\left(3a+3bx^2\right)\right)dx\\ &= \frac{1}{4}\int (ex)^m \sinh\left(3a+3bx^2\right)dx-\frac{3}{4}\int (ex)^m \sinh\left(a+bx^2\right)dx\\ &= -\left(\frac{1}{8}\int e^{-3a-3bx^2}(ex)^m\,dx\right)+\frac{1}{8}\int e^{3a+3bx^2}(ex)^m\,dx+\frac{3}{8}\int e^{-a-bx^2}(ex)^m\,dx-\frac{3}{8}\int e^{a+bx^2}(ex)^{m}\\ &= -\frac{3^{-\frac{1}{2}-\frac{m}{2}}e^{3a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-3bx^2\right)}{16e}+\frac{3e^a(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{16e}\end{aligned}$$

Mathematica [B] time = 12.7651, size = 735, normalized size = 3.43

$$\frac{1}{16}3^{\frac{1}{2}-\frac{m}{2}}x\sinh(a)\cosh^2(a)\left(-b^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left(\left(-bx^2\right)^{\frac{m+1}{2}}\left(3^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2},bx^2\right)-\text{Gamma}\left(\frac{m+1}{2},3bx^2\right)\right)-\left(b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^3,x]
```

```
[Out] ((e*x)^m*Cosh[a]^3*((-3*(-(x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/
2, -(b*x^2)]])/2 + (x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/
2))/8 + (-(3^((-1 - m)/2)*x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2
, -3*b*x^2])/2 + (3^((-1 - m)/2)*x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 +
m)/2, 3*b*x^2])/2)/8))/x^m + (3^(1/2 - m/2)*x*(e*x)^m*(-(b^2*x^4))^((-1 - m
)/2)*Cosh[a]^2*(-((b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -3*b*x^2]) + 3^((1 +
 m)/2)*(b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -(b*x^2)] + (-(b*x^2))^((1 + m)
/2)*(3^((1 + m)/2)*Gamma[(1 + m)/2, b*x^2] - Gamma[(1 + m)/2, 3*b*x^2]))*Si
nh[a])/16 - (3^(1/2 - m/2)*x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*Cosh[a]*((b*
x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -3*b*x^2] + 3^((1 + m)/2)*(b*x^2)^((1 + m
)/2)*Gamma[(1 + m)/2, -(b*x^2)] - (-(b*x^2))^((1 + m)/2)*(3^((1 + m)/2)*Gam
```

```
ma[(1 + m)/2, b*x^2] + Gamma[(1 + m)/2, 3*b*x^2]))*Sinh[a]^2)/16 + ((e*x)^m
*((3*(-(x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -(b*x^2)])/2 - (
x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/2))/8 + (-(3^((-1 -
 m)/2)*x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -3*b*x^2])/2 - (3
^((-1 - m)/2)*x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, 3*b*x^2])/2)/
8)*Sinh[a]^3)/x^m
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh\left(bx^2+a\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a)^3,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2+a\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^3, x)
```

Fricas [A] time = 1.929, size = 745, normalized size = 3.48

$$\frac{e\cosh\left(\frac{1}{2}(m-1)\log\left(\frac{3\,b}{e^2}\right)+3\,a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},3\,bx^2\right)-9\,e\cosh\left(\frac{1}{2}(m-1)\log\left(\frac{b}{e^2}\right)+a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},bx^2\right)-9\,e\cosh\left(\frac{1}{2}(m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(e*cosh(1/2*(m - 1)*log(3*b/e^2) + 3*a)*gamma(1/2*m + 1/2, 3*b*x^2) -
9*e*cosh(1/2*(m - 1)*log(b/e^2) + a)*gamma(1/2*m + 1/2, b*x^2) - 9*e*cosh(1
/2*(m - 1)*log(-b/e^2) - a)*gamma(1/2*m + 1/2, -b*x^2) + e*cosh(1/2*(m - 1)
*log(-3*b/e^2) - 3*a)*gamma(1/2*m + 1/2, -3*b*x^2) - e*gamma(1/2*m + 1/2, 3
*b*x^2)*sinh(1/2*(m - 1)*log(3*b/e^2) + 3*a) + 9*e*gamma(1/2*m + 1/2, b*x^2
)*sinh(1/2*(m - 1)*log(b/e^2) + a) + 9*e*gamma(1/2*m + 1/2, -b*x^2)*sinh(1/
2*(m - 1)*log(-b/e^2) - a) - e*gamma(1/2*m + 1/2, -3*b*x^2)*sinh(1/2*(m - 1
)*log(-3*b/e^2) - 3*a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^3\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2 + a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^3, x)
```

3.25 $\int (ex)^m \sinh^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=135

$$-\frac{e^{2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-2bx^2\right)}{e}-\frac{e^{-2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},2bx^2\right)}{e}-\frac{(ex}{2e(r}$$

```
[Out] -(e*x)^(1 + m)/(2*e*(1 + m)) - (2^(-7/2 - m/2)*E^(2*a)*(e*x)^(1 + m)*(-(b*x
^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -2*b*x^2])/e - (2^(-7/2 - m/2)*(e*x)^(1
+ m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, 2*b*x^2])/(e*E^(2*a))
```

Rubi [A] time = 0.147643, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5340, 5329, 2218}

$$-\frac{e^{2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-2bx^2\right)}{e}-\frac{e^{-2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},2bx^2\right)}{e}-\frac{(ex}{2e(r}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^2,x]
```

```
[Out] -(e*x)^(1 + m)/(2*e*(1 + m)) - (2^(-7/2 - m/2)*E^(2*a)*(e*x)^(1 + m)*(-(b*x
^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -2*b*x^2])/e - (2^(-7/2 - m/2)*(e*x)^(1
+ m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, 2*b*x^2])/(e*E^(2*a))
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5329

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^2\left(a+bx^2\right)\,dx &= \int \left(-\frac{1}{2}(ex)^m + \frac{1}{2}(ex)^m \cosh\left(2a+2bx^2\right)\right)dx\\ &= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{2}\int (ex)^m \cosh\left(2a+2bx^2\right)dx\\ &= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{4}\int e^{-2a-2bx^2}(ex)^m\,dx + \frac{1}{4}\int e^{2a+2bx^2}(ex)^m\,dx\\ &= -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{2a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2bx^2\right)}{e} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{-2a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(\cdots}}{e}\end{aligned}$$

Mathematica [A] time = 0.615218, size = 152, normalized size = 1.13

$$-\frac{2^{\frac{1}{2}(-m-7)}x\left(-b^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left((m+1)(\cosh(2a)-\sinh(2a))\left(-bx^2\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2},2bx^2\right)+(m+1)(\sinh(2a)+\text{co}\cdots\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^2,x]
```

```
[Out] -((2^((-7 - m)/2)*x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*(2^((5 + m)/2)*(-(b^2
*x^4))^((1 + m)/2) + (1 + m)*(-(b*x^2))^((1 + m)/2)*Gamma[(1 + m)/2, 2*b*x^
2]*(Cosh[2*a] - Sinh[2*a]) + (1 + m)*(b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -
2*b*x^2]*(Cosh[2*a] + Sinh[2*a])))/(1 + m))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (ex)^m\left(\sinh\left(bx^2+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a)^2,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83331, size = 512, normalized size = 3.79

$$-\frac{8\,bx\cosh\left(m\log\left(ex\right)\right) + \left(em+e\right)\cosh\left(\frac{1}{2}\left(m-1\right)\log\left(\frac{2\,b}{e^2}\right)+2\,a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2}, 2\,bx^2\right) - \left(em+e\right)\cosh\left(\frac{1}{2}\left(m-1\right)\log\left(-\frac{2}{e}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*b*x*cosh(m*log(e*x)) + (e*m + e)*cosh(1/2*(m - 1)*log(2*b/e^2) + 2
*a)*gamma(1/2*m + 1/2, 2*b*x^2) - (e*m + e)*cosh(1/2*(m - 1)*log(-2*b/e^2)
- 2*a)*gamma(1/2*m + 1/2, -2*b*x^2) + 8*b*x*sinh(m*log(e*x)) - (e*m + e)*ga
mma(1/2*m + 1/2, 2*b*x^2)*sinh(1/2*(m - 1)*log(2*b/e^2) + 2*a) + (e*m + e)*
gamma(1/2*m + 1/2, -2*b*x^2)*sinh(1/2*(m - 1)*log(-2*b/e^2) - 2*a))/(b*m +
b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex\right)^{m}\sinh^{2}\left(a+bx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2 + a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^2, x)
```

3.26 $\int (ex)^m \sinh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=95

$$\frac{e^{-a}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},bx^2\right)}{4e}-\frac{e^{a}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)}{4e}$$

```
[Out] -(E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -(b*x^2)])/(4*
e) + ((e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/(4*e*E^a)
```

Rubi [A] time = 0.0676833, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5328, 2218}

$$\frac{e^{-a}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},bx^2\right)}{4e}-\frac{e^{a}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2],x]
```

```
[Out] -(E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -(b*x^2)])/(4*
e) + ((e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/(4*e*E^a)
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (ex)^m \sinh\left(a+bx^2\right) dx = -\left(\frac{1}{2}\int e^{-a-bx^2}(ex)^m\, dx\right) + \frac{1}{2}\int e^{a+bx^2}(ex)^m\, dx$$
$$= -\frac{e^a(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{4e} + \frac{e^{-a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},bx^2\right)}{4e}$$

Mathematica [A] time = 0.153106, size = 98, normalized size = 1.03

$$-\frac{1}{4}x\left(-b^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left((\sinh(a)+\cosh(a))\left(bx^2\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)-(\cosh(a)-\sinh(a))\left(-bx^2\right)^{\frac{m+1}{2}}\text{Gamm}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2],x]
```

```
[Out] -(x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*(-((-(b*x^2))^((1 + m)/2)*Gamma[(1 +
m)/2, b*x^2]*(Cosh[a] - Sinh[a])) + (b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -(
b*x^2)]*(Cosh[a] + Sinh[a])))/4
```

Maple [C] time = 0.069, size = 77, normalized size = 0.8

$$\frac{(ex)^m x\sinh(a)}{1+m}{}_1F_2\left(\frac{m}{4}+\frac{1}{4};\frac{1}{2},\frac{5}{4}+\frac{m}{4};\frac{x^4b^2}{4}\right)+\frac{(ex)^m bx^3\cosh(a)}{3+m}{}_1F_2\left(\frac{3}{4}+\frac{m}{4};\frac{3}{2},\frac{7}{4}+\frac{m}{4};\frac{x^4b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a),x)
```

```
[Out] (e*x)^m/(1+m)*x*hypergeom([1/4*m+1/4],[1/2,5/4+1/4*m],1/4*x^4*b^2)*sinh(a)+
(e*x)^m*b/(3+m)*x^3*hypergeom([3/4+1/4*m],[3/2,7/4+1/4*m],1/4*x^4*b^2)*cosh
(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a), x)
```

Fricas [A] time = 1.77891, size = 355, normalized size = 3.74

$$\frac{e\cosh\left(\frac{1}{2}(m-1)\log\left(\frac{b}{e^2}\right)+a\right)\Gamma\left(\frac{1}{2}m+\frac{1}{2},bx^2\right)+e\cosh\left(\frac{1}{2}(m-1)\log\left(-\frac{b}{e^2}\right)-a\right)\Gamma\left(\frac{1}{2}m+\frac{1}{2},-bx^2\right)-e\Gamma\left(\frac{1}{2}m+\frac{1}{2},bx^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(e*cosh(1/2*(m - 1)*log(b/e^2) + a)*gamma(1/2*m + 1/2, b*x^2) + e*cosh(
1/2*(m - 1)*log(-b/e^2) - a)*gamma(1/2*m + 1/2, -b*x^2) - e*gamma(1/2*m + 1
/2, b*x^2)*sinh(1/2*(m - 1)*log(b/e^2) + a) - e*gamma(1/2*m + 1/2, -b*x^2)*
sinh(1/2*(m - 1)*log(-b/e^2) - a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a), x)
```

3.27 $\int (ex)^m \mathbf{csch}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=25

$$x^{-m}(ex)^m\text{Unintegrable}\left(x^m\text{csch}\left(a+bx^2\right),x\right)$$

```
[Out] ((e*x)^m*Unintegrable[x^m*Csch[a + b*x^2], x])/x^m
```

Rubi [A] time = 0.0251673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m\text{csch}\left(a+bx^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b*x^2],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b*x^2], x])/x^m
```

Rubi steps

$$\int (ex)^m\text{csch}\left(a+bx^2\right)\,dx=\left(x^{-m}(ex)^m\right)\int x^m\text{csch}\left(a+bx^2\right)\,dx$$

Mathematica [A] time = 2.71129, size = 0, normalized size = 0.

$$\int (ex)^m\text{csch}\left(a+bx^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b*x^2],x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b*x^2], x]
```

Maple [A] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(b*x^2+a),x)
```

```
[Out] int((e*x)^m/sinh(b*x^2+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/sinh(b*x^2 + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex)^m}{\sinh\left(bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/sinh(b*x^2 + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(b*x**2+a),x)
```

```
[Out] Integral((e*x)**m/sinh(a + b*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(bx^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(b*x^2 + a), x)
```

3.28 $\int x^3 \sinh\left(a + bx^4\right) dx$

Optimal. Leaf size=15

$$\frac{\cosh\left(a + bx^4\right)}{4b}$$

```
[Out] Cosh[a + b*x^4]/(4*b)
```

Rubi [A] time = 0.0203076, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$\frac{\cosh\left(a + bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^4],x]
```

```
[Out] Cosh[a + b*x^4]/(4*b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3 \sinh\left(a+bx^4\right) dx = \frac{1}{4}\,\text{Subst}\left(\int \sinh(a+bx)\,dx, x, x^4\right)$$
$$= \frac{\cosh\left(a+bx^4\right)}{4b}$$

Mathematica [A] time = 0.0082737, size = 15, normalized size = 1.

$$\frac{\cosh\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^4],x]
```

```
[Out] Cosh[a + b*x^4]/(4*b)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\cosh\left(bx^4+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^4+a),x)
```

```
[Out] 1/4*cosh(b*x^4+a)/b
```

Maxima [A] time = 1.03286, size = 18, normalized size = 1.2

$$\frac{\cosh\left(bx^4+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*cosh(b*x^4 + a)/b
```

Fricas [A] time = 1.7345, size = 31, normalized size = 2.07

$$\frac{\cosh\left(bx^4 + a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*cosh(b*x^4 + a)/b
```

Sympy [A] time = 1.14299, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\cosh\left(a+bx^4\right)}{4b} & \text{for}\ b \neq 0 \\ \frac{x^4 \sinh(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**4+a),x)
```

```
[Out] Piecewise((cosh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*sinh(a)/4, True))
```

Giac [A] time = 1.23275, size = 34, normalized size = 2.27

$$\frac{e^{\left(bx^4+a\right)} + e^{\left(-bx^4-a\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(e^(b*x^4 + a) + e^(-b*x^4 - a))/b
```

3.29 $\int x^2 \sinh\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=78

$$-\frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)$$

```
[Out] (b*x^2*Cosh[a + b/x])/6 - (b^3*Cosh[a]*CoshIntegral[b/x])/6 + (b^2*x*Sinh[a
 + b/x])/6 + (x^3*Sinh[a + b/x])/3 - (b^3*Sinh[a]*SinhIntegral[b/x])/6
```

Rubi [A] time = 0.143453, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b/x],x]
```

```
[Out] (b*x^2*Cosh[a + b/x])/6 - (b^3*Cosh[a]*CoshIntegral[b/x])/6 + (b^2*x*Sinh[a
 + b/x])/6 + (x^3*Sinh[a + b/x])/3 - (b^3*Sinh[a]*SinhIntegral[b/x])/6
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sinh\left(a+\frac{b}{x}\right)dx &= -\operatorname{Subst}\left(\int\frac{\sinh(a+bx)}{x^4}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)-\frac{1}{3}b\operatorname{Subst}\left(\int\frac{\cosh(a+bx)}{x^3}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)-\frac{1}{6}b^2\operatorname{Subst}\left(\int\frac{\sinh(a+bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\operatorname{Subst}\left(\int\frac{\cosh(a+bx)}{x}\,dx,\right.\\ &= \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)-\frac{1}{6}\left(b^3\cosh(a)\right)\operatorname{Subst}\left(\int\frac{\cosh(b}{x}\right.\\ &= \frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\cosh(a)\operatorname{Chi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(\end{aligned}$$

Mathematica [A] time = 0.0649958, size = 70, normalized size = 0.9

$$\frac{1}{6}\left(b^3(-\cosh(a))\operatorname{Chi}\left(\frac{b}{x}\right)-b^3\sinh(a)\operatorname{Shi}\left(\frac{b}{x}\right)+x\left(b^2\sinh\left(a+\frac{b}{x}\right)+2x^2\sinh\left(a+\frac{b}{x}\right)+bx\cosh\left(a+\frac{b}{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b/x],x]
```

```
[Out] (-(b^3*Cosh[a]*CoshIntegral[b/x]) + x*(b*x*Cosh[a + b/x] + b^2*Sinh[a + b/x
] + 2*x^2*Sinh[a + b/x]) - b^3*Sinh[a]*SinhIntegral[b/x])/6
```

Maple [A] time = 0.036, size = 130, normalized size = 1.7

$$-\frac{b^2x}{12}e^{-\frac{ax+b}{x}}+\frac{bx^2}{12}e^{-\frac{ax+b}{x}}-\frac{x^3}{6}e^{-\frac{ax+b}{x}}+\frac{b^3e^{-a}}{12}\mathrm{Ei}\left(1,\frac{b}{x}\right)+\frac{x^3}{6}e^{\frac{ax+b}{x}}+\frac{bx^2}{12}e^{\frac{ax+b}{x}}+\frac{b^2x}{12}e^{\frac{ax+b}{x}}+\frac{b^3e^{a}}{12}\mathrm{Ei}\left(1,-\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b/x),x)
```

```
[Out] -1/12*b^2*exp(-(a*x+b)/x)*x+1/12*b*exp(-(a*x+b)/x)*x^2-1/6*exp(-(a*x+b)/x)*
x^3+1/12*b^3*exp(-a)*Ei(1,b/x)+1/6*exp((a*x+b)/x)*x^3+1/12*b*exp((a*x+b)/x)
*x^2+1/12*b^2*exp((a*x+b)/x)*x+1/12*b^3*exp(a)*Ei(1,-b/x)
```

Maxima [A] time = 1.16563, size = 63, normalized size = 0.81

$$\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}\left(b^2e^{(-a)}\Gamma\left(-2,\frac{b}{x}\right)+b^2e^{a}\Gamma\left(-2,-\frac{b}{x}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(a + b/x) + 1/6*(b^2*e^(-a)*gamma(-2, b/x) + b^2*e^a*gamma(-2,
-b/x))*b
```

Fricas [A] time = 1.7202, size = 212, normalized size = 2.72

$$\frac{1}{6}bx^2\cosh\left(\frac{ax+b}{x}\right)-\frac{1}{12}\left(b^3\mathrm{Ei}\left(\frac{b}{x}\right)+b^3\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a)-\frac{1}{12}\left(b^3\mathrm{Ei}\left(\frac{b}{x}\right)-b^3\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)+\frac{1}{6}\left(b^2x+2x^3\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^2*cosh((a*x + b)/x) - 1/12*(b^3*Ei(b/x) + b^3*Ei(-b/x))*cosh(a) - 1
/12*(b^3*Ei(b/x) - b^3*Ei(-b/x))*sinh(a) + 1/6*(b^2*x + 2*x^3)*sinh((a*x +
b)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b/x),x)
```

```
[Out] Integral(x**2*sinh(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(a + b/x), x)
```

3.30 $\int x \sinh\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=60

$$-\frac{1}{2}b^2\sinh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)$$

```
[Out] (b*x*Cosh[a + b/x])/2 - (b^2*CoshIntegral[b/x]*Sinh[a])/2 + (x^2*Sinh[a + b
/x])/2 - (b^2*Cosh[a]*SinhIntegral[b/x])/2
```

Rubi [A] time = 0.107096, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b^2\sinh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b/x],x]
```

```
[Out] (b*x*Cosh[a + b/x])/2 - (b^2*CoshIntegral[b/x]*Sinh[a])/2 + (x^2*Sinh[a + b
/x])/2 - (b^2*Cosh[a]*SinhIntegral[b/x])/2
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sinh\left(a+\frac{b}{x}\right)dx &= -\text{Subst}\left(\int\frac{\sinh(a+bx)}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}b\,\text{Subst}\left(\int\frac{\cosh(a+bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}b^2\,\text{Subst}\left(\int\frac{\sinh(a+bx)}{x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}\left(b^2\cosh(a)\right)\text{Subst}\left(\int\frac{\sinh(bx)}{x}\,dx,x,\frac{1}{x}\right)-\frac{1}{2}\left(b^2\sin\right.\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)-\frac{1}{2}b^2\text{Chi}\left(\frac{b}{x}\right)\sinh(a)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0440879, size = 54, normalized size = 0.9

$$\frac{1}{2}\left(b^2\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right)-b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)+x\left(x\sinh\left(a+\frac{b}{x}\right)+b\cosh\left(a+\frac{b}{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b/x],x]
```

```
[Out] (-(b^2*CoshIntegral[b/x]*Sinh[a]) + x*(b*Cosh[a + b/x] + x*Sinh[a + b/x]) -
 b^2*Cosh[a]*SinhIntegral[b/x])/2
```

Maple [A] time = 0.03, size = 93, normalized size = 1.6

$$\frac{bx}{4}e^{-\frac{ax+b}{x}}-\frac{x^2}{4}e^{-\frac{ax+b}{x}}-\frac{b^2e^{-a}}{4}\mathrm{Ei}\left(1,\frac{b}{x}\right)+\frac{x^2}{4}e^{\frac{ax+b}{x}}+\frac{bx}{4}e^{\frac{ax+b}{x}}+\frac{b^2e^{a}}{4}\mathrm{Ei}\left(1,-\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b/x),x)
```

```
[Out] 1/4*b*exp(-(a*x+b)/x)*x-1/4*exp(-(a*x+b)/x)*x^2-1/4*b^2*exp(-a)*Ei(1,b/x)+1
/4*exp((a*x+b)/x)*x^2+1/4*b*exp((a*x+b)/x)*x+1/4*b^2*exp(a)*Ei(1,-b/x)
```

Maxima [A] time = 1.15938, size = 59, normalized size = 0.98

$$\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)+\frac{1}{4}\left(be^{(-a)}\Gamma\left(-1,\frac{b}{x}\right)-be^{a}\Gamma\left(-1,-\frac{b}{x}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh(a + b/x) + 1/4*(b*e^(-a)*gamma(-1, b/x) - b*e^a*gamma(-1, -b/x
))*b
```

Fricas [A] time = 1.72197, size = 190, normalized size = 3.17

$$\frac{1}{2}bx\cosh\left(\frac{ax+b}{x}\right)+\frac{1}{2}x^2\sinh\left(\frac{ax+b}{x}\right)-\frac{1}{4}\left(b^2\mathrm{Ei}\left(\frac{b}{x}\right)-b^2\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a)-\frac{1}{4}\left(b^2\mathrm{Ei}\left(\frac{b}{x}\right)+b^2\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/2*b*x*cosh((a*x + b)/x) + 1/2*x^2*sinh((a*x + b)/x) - 1/4*(b^2*Ei(b/x) -
b^2*Ei(-b/x))*cosh(a) - 1/4*(b^2*Ei(b/x) + b^2*Ei(-b/x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x),x)
```

```
[Out] Integral(x*sinh(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(a + b/x), x)
```

3.31 $\int \sinh\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=33

$$-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)$$

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

Rubi [A] time = 0.0757435, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5302, 3297, 3303, 3298, 3301}

$$-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh\left(a+\frac{b}{x}\right)dx &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= x\sinh\left(a+\frac{b}{x}\right)-b\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= x\sinh\left(a+\frac{b}{x}\right)-(b\cosh(a))\,\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,\frac{1}{x}\right)-(b\sinh(a))\,\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx,x,\right.\\ &= -b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0196536, size = 33, normalized size = 1.

$$-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

Maple [A] time = 0.029, size = 56, normalized size = 1.7

$$\frac{b\mathrm{e}^{-a}}{2}\mathrm{Ei}\left(1,\frac{b}{x}\right)-\frac{x}{2}\mathrm{e}^{-\frac{ax+b}{x}}+\frac{\mathrm{e}^{a}b}{2}\mathrm{Ei}\left(1,-\frac{b}{x}\right)+\frac{x}{2}\mathrm{e}^{\frac{ax+b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x),x)
```

```
[Out] 1/2*b*exp(-a)*Ei(1,b/x)-1/2*exp(-(a*x+b)/x)*x+1/2*b*exp(a)*Ei(1,-b/x)+1/2*e
xp((a*x+b)/x)*x
```

Maxima [A] time = 1.15812, size = 49, normalized size = 1.48

$$-\frac{1}{2}\left(\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)}+\mathrm{Ei}\left(\frac{b}{x}\right)e^{a}\right)b+x\sinh\left(a+\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x, algorithm="maxima")
```

```
[Out] -1/2*(Ei(-b/x)*e^(-a) + Ei(b/x)*e^a)*b + x*sinh(a + b/x)
```

Fricas [A] time = 1.6999, size = 135, normalized size = 4.09

$$-\frac{1}{2}\left(b\mathrm{Ei}\left(\frac{b}{x}\right)+b\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh\left(a\right)-\frac{1}{2}\left(b\mathrm{Ei}\left(\frac{b}{x}\right)-b\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh\left(a\right)+x\sinh\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x, algorithm="fricas")
```

```
[Out] -1/2*(b*Ei(b/x) + b*Ei(-b/x))*cosh(a) - 1/2*(b*Ei(b/x) - b*Ei(-b/x))*sinh(a
) + x*sinh((a*x + b)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x)
```

```
[Out] Integral(sinh(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x), x)
```

3.32 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal. Leaf size=21

$$\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right)-\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

Rubi [A] time = 0.031971, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5318, 5317, 5316}

$$\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right)-\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x,x]
```

```
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = \cosh(a)\int \frac{\sinh\left(\frac{b}{x}\right)}{x}\,dx + \sinh(a)\int \frac{\cosh\left(\frac{b}{x}\right)}{x}\,dx$$
$$= -\mathrm{Chi}\left(\frac{b}{x}\right)\sinh(a) - \cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)$$

Mathematica [A] time = 0.0147595, size = 21, normalized size = 1.

$$\sinh(a)\left(-\mathrm{Chi}\left(\frac{b}{x}\right)\right) - \cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x,x]
```

```
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

Maple [A] time = 0.022, size = 27, normalized size = 1.3

$$-\frac{e^{-a}}{2}\mathrm{Ei}\left(1,\frac{b}{x}\right) + \frac{e^{a}}{2}\mathrm{Ei}\left(1,-\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x,x)
```

```
[Out] -1/2*exp(-a)*Ei(1,b/x)+1/2*exp(a)*Ei(1,-b/x)
```

Maxima [A] time = 1.21361, size = 32, normalized size = 1.52

$$\frac{1}{2}\,\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)} - \frac{1}{2}\,\mathrm{Ei}\left(\frac{b}{x}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="maxima")
```

```
[Out] 1/2*Ei(-b/x)*e^(-a) - 1/2*Ei(b/x)*e^a
```

Fricas [A] time = 1.70971, size = 95, normalized size = 4.52

$$-\frac{1}{2}\left(\mathrm{Ei}\left(\frac{b}{x}\right)-\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh\left(a\right)-\frac{1}{2}\left(\mathrm{Ei}\left(\frac{b}{x}\right)+\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(Ei(b/x) - Ei(-b/x))*cosh(a) - 1/2*(Ei(b/x) + Ei(-b/x))*sinh(a)
```

Sympy [A] time = 1.67246, size = 17, normalized size = 0.81

$$-\sinh\left(a\right)\mathrm{Chi}\left(\frac{b}{x}\right)-\cosh\left(a\right)\mathrm{Shi}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x)
```

```
[Out] -sinh(a)*Chi(b/x) - cosh(a)*Shi(b/x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x)/x, x)
```

3.33 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=13

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -(Cosh[a + b/x]/b)
```

Rubi [A] time = 0.0169405, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^2,x]

[Out] -(Cosh[a + b/x]/b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\operatorname{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Mathematica [A] time = 0.0045134, size = 13, normalized size = 1.

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^2,x]

[Out] -(Cosh[a + b/x]/b)
```

Maple [A] time = 0.004, size = 14, normalized size = 1.1

$$-\frac{1}{b}\cosh\left(a+\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^2,x)

[Out] -cosh(a+b/x)/b
```

Maxima [A] time = 1.05249, size = 18, normalized size = 1.38

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] -cosh(a + b/x)/b
```

Fricas [A] time = 1.62389, size = 30, normalized size = 2.31

$$-\frac{\cosh\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] -cosh((a*x + b)/x)/b
```

Sympy [A] time = 1.56414, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\sinh\left(a\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**2,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/b, Ne(b, 0)), (-sinh(a)/x, True))
```

Giac [A] time = 1.25665, size = 34, normalized size = 2.62

$$-\frac{e^{\left(a+\frac{b}{x}\right)}+e^{\left(-a-\frac{b}{x}\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^(a + b/x) + e^(-a - b/x))/b
```

3.34 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}$$

```
[Out] -(Cosh[a + b/x]/(b*x)) + Sinh[a + b/x]/b^2
```

Rubi [A] time = 0.0302479, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 3296, 2637}

$$\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^3,x]

[Out] -(Cosh[a + b/x]/(b*x)) + Sinh[a + b/x]/b^2
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx} + \frac{\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx} + \frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}$$

Mathematica [A] time = 0.0255047, size = 29, normalized size = 1.

$$\frac{x\sinh\left(a+\frac{b}{x}\right)-b\cosh\left(a+\frac{b}{x}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^3,x]
```

```
[Out] (-(b*Cosh[a + b/x]) + x*Sinh[a + b/x])/(b^2*x)
```

Maple [A] time = 0.009, size = 44, normalized size = 1.5

$$-\frac{1}{b^2}\left(\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)-a\cosh\left(a+\frac{b}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^3,x)
```

```
[Out] -1/b^2*((a+b/x)*cosh(a+b/x)-sinh(a+b/x)-a*cosh(a+b/x))
```

Maxima [C] time = 1.18438, size = 65, normalized size = 2.24

$$-\frac{1}{4}b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x}\right)}{b^3}-\frac{e^{a}\Gamma\left(3,-\frac{b}{x}\right)}{b^3}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(e^(-a)*gamma(3, b/x)/b^3 - e^a*gamma(3, -b/x)/b^3) - 1/2*sinh(a + b
/x)/x^2
```

Fricas [A] time = 1.63522, size = 73, normalized size = 2.52

$$-\frac{b\cosh\left(\frac{ax+b}{x}\right)-x\sinh\left(\frac{ax+b}{x}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] -(b*cosh((a*x + b)/x) - x*sinh((a*x + b)/x))/(b^2*x)
```

Sympy [A] time = 3.08758, size = 29, normalized size = 1.

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}+\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b \neq 0 \\ -\frac{\sinh(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**3,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x) + sinh(a + b/x)/b**2, Ne(b, 0)), (-sinh(a)/
(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x)/x^3, x)
```

3.35 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[Out] (-2*Cosh[a + b/x])/b^3 - Cosh[a + b/x]/(b*x^2) + (2*Sinh[a + b/x])/(b^2*x)
```

Rubi [A] time = 0.0538775, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 3296, 2638}

$$\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^4,x]
```

```
[Out] (-2*Cosh[a + b/x])/b^3 - Cosh[a + b/x]/(b*x^2) + (2*Sinh[a + b/x])/(b^2*x)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\,\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\ &= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\,\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\ &= -\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.0439151, size = 39, normalized size = 0.85

$$\frac{2bx\sinh\left(a+\frac{b}{x}\right)-\left(b^2+2x^2\right)\cosh\left(a+\frac{b}{x}\right)}{b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^4,x]
```

```
[Out] (-((b^2 + 2*x^2)*Cosh[a + b/x]) + 2*b*x*Sinh[a + b/x])/(b^3*x^2)
```

Maple [B] time = 0.007, size = 94, normalized size = 2.

$$-\frac{1}{b^3}\left(\left(a+\frac{b}{x}\right)^2\cosh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)+2\cosh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^4,x)
```

```
[Out] -1/b^3*((a+b/x)^2*cosh(a+b/x)-2*(a+b/x)*sinh(a+b/x)+2*cosh(a+b/x)-2*a*((a+b
/x)*cosh(a+b/x)-sinh(a+b/x))+a^2*cosh(a+b/x))
```

Maxima [C] time = 1.25092, size = 63, normalized size = 1.37

$$-\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x}\right)}{b^4}+\frac{e^a\Gamma\left(4,-\frac{b}{x}\right)}{b^4}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(e^(-a)*gamma(4, b/x)/b^4 + e^a*gamma(4, -b/x)/b^4) - 1/3*sinh(a + b
/x)/x^3
```

Fricas [A] time = 1.71541, size = 96, normalized size = 2.09

$$\frac{2\,bx\sinh\left(\frac{ax+b}{x}\right)-\left(b^2+2\,x^2\right)\cosh\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] (2*b*x*sinh((a*x + b)/x) - (b^2 + 2*x^2)*cosh((a*x + b)/x))/(b^3*x^2)
```

Sympy [A] time = 5.23308, size = 46, normalized size = 1.

$$\begin{cases}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b\neq 0\\ -\frac{\sinh(a)}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**4,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x**2) + 2*sinh(a + b/x)/(b**2*x) - 2*cosh(a +
b/x)/b**3, Ne(b, 0)), (-sinh(a)/(3*x**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + \frac{b}{x}\right)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x)/x^4, x)
```

3.36 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx$

Optimal. Leaf size=62

$$\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}$$

```
[Out] -(Cosh[a + b/x]/(b*x^3)) - (6*Cosh[a + b/x])/(b^3*x) + (6*Sinh[a + b/x])/b^
4 + (3*Sinh[a + b/x])/(b^2*x^2)
```

Rubi [A] time = 0.0795417, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 3296, 2637}

$$\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^5,x]
```

```
[Out] -(Cosh[a + b/x]/(b*x^3)) - (6*Cosh[a + b/x])/(b^3*x) + (6*Sinh[a + b/x])/b^
4 + (3*Sinh[a + b/x])/(b^2*x^2)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx &= -\text{Subst}\left(\int x^3\sinh(a+bx)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}+\frac{3\,\text{Subst}\left(\int x^2\cosh(a+bx)\,dx,x,\frac{1}{x}\right)}{b}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}+\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}-\frac{6\,\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,\frac{1}{x}\right)}{b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}+\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{6\,\text{Subst}\left(\int\cosh(a+bx)\,dx,x,\frac{1}{x}\right)}{b^3}\\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4}+\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}
\end{aligned}$$

Mathematica [A] time = 0.058288, size = 48, normalized size = 0.77

$$\frac{3x\left(b^2+2x^2\right)\sinh\left(a+\frac{b}{x}\right)-b\left(b^2+6x^2\right)\cosh\left(a+\frac{b}{x}\right)}{b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^5,x]
```

```
[Out] (-(b*(b^2 + 6*x^2)*Cosh[a + b/x]) + 3*x*(b^2 + 2*x^2)*Sinh[a + b/x])/(b^4*x
^3)
```

Maple [B] time = 0.009, size = 165, normalized size = 2.7

$$-\frac{1}{b^4}\left(\left(a+\frac{b}{x}\right)^3\cosh\left(a+\frac{b}{x}\right)-3\left(a+\frac{b}{x}\right)^2\sinh\left(a+\frac{b}{x}\right)+6\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-6\sinh\left(a+\frac{b}{x}\right)-3a\left(\left(a+\frac{b}{x}\right)^2\cosh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^5,x)
```

```
[Out] -1/b^4*((a+b/x)^3*cosh(a+b/x)-3*(a+b/x)^2*sinh(a+b/x)+6*(a+b/x)*cosh(a+b/x)
-6*sinh(a+b/x)-3*a*((a+b/x)^2*cosh(a+b/x)-2*(a+b/x)*sinh(a+b/x)+2*cosh(a+b/
x))+3*a^2*((a+b/x)*cosh(a+b/x)-sinh(a+b/x))-a^3*cosh(a+b/x))
```

Maxima [C] time = 1.23791, size = 65, normalized size = 1.05

$$-\frac{1}{8}b\left(\frac{e^{(-a)}\Gamma\left(5,\frac{b}{x}\right)}{b^5}-\frac{e^a\Gamma\left(5,-\frac{b}{x}\right)}{b^5}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*(e^(-a)*gamma(5, b/x)/b^5 - e^a*gamma(5, -b/x)/b^5) - 1/4*sinh(a + b
/x)/x^4
```

Fricas [A] time = 1.7854, size = 116, normalized size = 1.87

$$-\frac{\left(b^3+6\,bx^2\right)\cosh\left(\frac{ax+b}{x}\right)-3\left(b^2x+2\,x^3\right)\sinh\left(\frac{ax+b}{x}\right)}{b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] -((b^3 + 6*b*x^2)*cosh((a*x + b)/x) - 3*(b^2*x + 2*x^3)*sinh((a*x + b)/x))/
(b^4*x^3)
```

Sympy [A] time = 9.01725, size = 61, normalized size = 0.98

$$\begin{cases}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}+\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4} & \text{for}\ b\neq 0\\ -\frac{\sinh(a)}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**5,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x**3) + 3*sinh(a + b/x)/(b**2*x**2) - 6*cosh(a
 + b/x)/(b**3*x) + 6*sinh(a + b/x)/b**4, Ne(b, 0)), (-sinh(a)/(4*x**4), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x)/x^5, x)
```

3.37 $\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=146

$$-\frac{1}{8}e^{3a}b3^{m+1}\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{3b}{x}\right)+\frac{3}{8}e^ab\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{b}{x}\right)+\frac{3}{8}e^{-a}b\left(\frac{b}{x}\right)^m(ex)^m\text{Gam}$$

```
[Out] -(3^(1 + m)*b*E^(3*a)*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, (-3*b)/x])/8 + (3*b*
E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/8 + (3*b*(b/x)^m*(e*x)^m*Gamm
a[-1 - m, b/x])/(8*E^a) - (3^(1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (3*b)/
x])/(8*E^(3*a))
```

Rubi [A] time = 0.250817, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5350, 3312, 3308, 2181}

$$-\frac{1}{8}e^{3a}b3^{m+1}\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{3b}{x}\right)+\frac{3}{8}e^ab\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{b}{x}\right)+\frac{3}{8}e^{-a}b\left(\frac{b}{x}\right)^m(ex)^m\text{Gam}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x]^3,x]
```


```
[Out] -(3^(1 + m)*b*E^(3*a)*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, (-3*b)/x])/8 + (3*b*
E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/8 + (3*b*(b/x)^m*(e*x)^m*Gamm
a[-1 - m, b/x])/(8*E^a) - (3^(1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (3*b)/
x])/(8*E^(3*a))
```


Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh^3(a+bx)\,dx, x, \frac{1}{x}\right)\right)\\ &= -\left(\left(i\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(\frac{3}{4}ix^{-2-m}\sinh(a+bx)-\frac{1}{4}ix^{-2-m}\sinh(3a+3bx)\right) dx, x, \frac{1}{x}\right)\right)\\ &= -\left(\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh(3a+3bx)\,dx, x, \frac{1}{x}\right)\right)+\frac{1}{4}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int\right.\\ &= -\left(\frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(3ia+3ibx)}x^{-2-m}\,dx, x, \frac{1}{x}\right)\right)+\frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{i(3ia+}\right.\\ &= -\frac{1}{8}3^{1+m}be^{3a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{3b}{x}\right)+\frac{3}{8}be^{a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{b}{x}\right)+\frac{3}{8}be^{-a}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x]^3,x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh\left(a+\frac{b}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x)^3,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \sinh\left(\frac{ax+b}{x}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x + b)/x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + \frac{b}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x)^3, x)
```

3.38 $\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=90

$$-e^{2a}b2^{m-1}\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{2b}{x}\right)+e^{-2a}b2^{m-1}\left(\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,\frac{2b}{x}\right)-\frac{x(ex)^m}{2(m+1)}$$

```
[Out] -(x*(e*x)^m)/(2*(1 + m)) - 2^(-1 + m)*b*E^(2*a)*(-(b/x))^m*(e*x)^m*Gamma[-1
 - m, (-2*b)/x] + (2^(-1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (2*b)/x])/E^(
2*a)
```

Rubi [A] time = 0.157582, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5350, 3312, 3307, 2181}

$$-e^{2a}b2^{m-1}\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{2b}{x}\right)+e^{-2a}b2^{m-1}\left(\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,\frac{2b}{x}\right)-\frac{x(ex)^m}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x]^2,x]
```

```
[Out] -(x*(e*x)^m)/(2*(1 + m)) - 2^(-1 + m)*b*E^(2*a)*(-(b/x))^m*(e*x)^m*Gamma[-1
 - m, (-2*b)/x] + (2^(-1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (2*b)/x])/E^(
2*a)
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh^2(a+bx)\,dx, x, \frac{1}{x}\right)\right)\\
&= \left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(\frac{x^{-2-m}}{2} - \frac{1}{2}x^{-2-m}\cosh(2a+2bx)\right) dx, x, \frac{1}{x}\right)\\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\cosh(2a+2bx)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(2ia+2ibx)}x^{-2-m}\,dx, x, \frac{1}{x}\right) - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\right.\\
&= -\frac{x(ex)^m}{2(1+m)} - 2^{-1+m}be^{2a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{2b}{x}\right) + 2^{-1+m}be^{-2a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,\frac{2b}{x}\right.
\end{aligned}$$

Mathematica [A] time = 0.25195, size = 88, normalized size = 0.98

$$-\frac{(ex)^m\left(b2^m(m+1)(\sinh(a)+\cosh(a))^2\left(-\frac{b}{x}\right)^m \text{Gamma}\left(-m-1,-\frac{2b}{x}\right) - b2^m(m+1)(\cosh(a)-\sinh(a))^2\left(\frac{b}{x}\right)^m \text{Gamma}\right.}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x]^2,x]
```

```
[Out] -((e*x)^m*(x - 2^m*b*(1 + m)*(b/x)^m*Gamma[-1 - m, (2*b)/x]*(Cosh[a] - Sinh
[a])^2 + 2^m*b*(1 + m)*(-(b/x))^m*Gamma[-1 - m, (-2*b)/x]*(Cosh[a] + Sinh[a
])^2))/(2*(1 + m))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh \left(a+\frac{b}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x)^2,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax+b}{x}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x + b)/x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + \frac{b}{x}\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x)^2, x)
```

3.39 $\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}e^a b\left(-\frac{b}{x}\right)^m (ex)^m \text{Gamma}\left(-m-1,-\frac{b}{x}\right)-\frac{1}{2}e^{-a} b\left(\frac{b}{x}\right)^m (ex)^m \text{Gamma}\left(-m-1,\frac{b}{x}\right)$$

```
[Out] -(b*E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/2 - (b*(b/x)^m*(e*x)^m*Ga
mma[-1 - m, b/x])/(2*E^a)
```

Rubi [A] time = 0.0875178, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5350, 3308, 2181}

$$-\frac{1}{2}e^a b\left(-\frac{b}{x}\right)^m (ex)^m \text{Gamma}\left(-m-1,-\frac{b}{x}\right)-\frac{1}{2}e^{-a} b\left(\frac{b}{x}\right)^m (ex)^m \text{Gamma}\left(-m-1,\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x],x]
```

```
[Out] -(b*E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/2 - (b*(b/x)^m*(e*x)^m*Ga
mma[-1 - m, b/x])/(2*E^a)
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh\left(a+\frac{b}{x}\right)dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh(a+bx)\,dx, x, \frac{1}{x}\right)\right)\\ &= -\left(\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{-i(ia+ibx)}x^{-2-m}\,dx, x, \frac{1}{x}\right)\right)+\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{i(ia+ibx)}x\right.\\ &= -\frac{1}{2}be^a\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{b}{x}\right)-\frac{1}{2}be^{-a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0752309, size = 63, normalized size = 0.94

$$-\frac{1}{2}b(ex)^m\left((\sinh(a)+\cosh(a))\left(-\frac{b}{x}\right)^m \text{Gamma}\left(-m-1,-\frac{b}{x}\right)+(\cosh(a)-\sinh(a))\left(\frac{b}{x}\right)^m \text{Gamma}\left(-m-1,\frac{b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x],x]
```

```
[Out] -(b*(e*x)^m*((b/x)^m*Gamma[-1 - m, b/x]*(Cosh[a] - Sinh[a]) + (-(b/x))^m*Ga
mma[-1 - m, -(b/x)]*(Cosh[a] + Sinh[a])))/2
```

Maple [C] time = 0.034, size = 70, normalized size = 1.

$$\frac{(ex)^m b\cosh(a)}{m}{}_1F_2(-\frac{m}{2};\frac{3}{2},1-\frac{m}{2};\frac{b^2}{4x^2})+\frac{(ex)^m x\sinh(a)}{1+m}{}_1F_2(-\frac{1}{2}-\frac{m}{2};\frac{1}{2},\frac{1}{2}-\frac{m}{2};\frac{b^2}{4x^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x),x)
```

```
[Out] (e*x)^m*b/m*hypergeom([-1/2*m],[3/2,1-1/2*m],1/4/x^2*b^2)*cosh(a)+(e*x)^m/(
1+m)*x*hypergeom([-1/2-1/2*m],[1/2,1/2-1/2*m],1/4/x^2*b^2)*sinh(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="maxima")

[Out] integrate((e*x)^m*sinh(a + b/x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax+b}{x}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="fricas")

[Out] integral((e*x)^m*sinh((a*x + b)/x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x),x)

[Out] Integral((e*x)**m*sinh(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x), x)
```

3.40 $\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=25

$$x^{-m}(ex)^m \text{Unintegrable}\left(x^m \operatorname{csch}\left(a+\frac{b}{x}\right),x\right)$$

```
[Out] ((e*x)^m*Unintegrable[x^m*Csch[a + b/x], x])/x^m
```

Rubi [A] time = 0.0236974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b/x],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b/x], x])/x^m
```

Rubi steps

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx = \left(x^{-m}(ex)^m\right)\int x^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx$$

Mathematica [A] time = 3.08518, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b/x],x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b/x], x]
```

Maple [A] time = 0.032, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh \left(a+\frac{b}{x}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a+b/x),x)

[Out] int((e*x)^m/sinh(a+b/x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh \left(a+\frac{b}{x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="maxima")

[Out] integrate((e*x)^m/sinh(a + b/x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{\sinh \left(\frac{ax+b}{x}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="fricas")

[Out] integral((e*x)^m/sinh((a*x + b)/x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(a+b/x),x)
```

```
[Out] Integral((e*x)**m/sinh(a + b/x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(a + b/x), x)
```

3.41 $\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=104

$$-\frac{2}{15}\sqrt{\pi}e^{-a}b^{5/2}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{2}{15}\sqrt{\pi}e^{a}b^{5/2}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right)$$

```
[Out] (2*b*x^3*Cosh[a + b/x^2])/15 - (2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(15*E^a)
 - (2*b^(5/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/15 + (4*b^2*x*Sinh[a + b/x^2])/
15 + (x^5*Sinh[a + b/x^2])/5
```

Rubi [A] time = 0.0903547, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5346, 5326, 5327, 5299, 2204, 2205}

$$-\frac{2}{15}\sqrt{\pi}e^{-a}b^{5/2}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{2}{15}\sqrt{\pi}e^{a}b^{5/2}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x^3*Cosh[a + b/x^2])/15 - (2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(15*E^a)
 - (2*b^(5/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/15 + (4*b^2*x*Sinh[a + b/x^2])/
15 + (x^5*Sinh[a + b/x^2])/5
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5327

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*
x)^(m + 1)*Cosh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx = -\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^6} dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{\cosh\left(a+bx^2\right)}{x^4} dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}\left(4b^2\right)\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) + \frac{4}{15}b^2x \sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}\left(8b^3\right)\text{Subst}\left(\int \cosh\right.$$

$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) + \frac{4}{15}b^2x \sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}\left(4b^3\right)\text{Subst}\left(\int e^{-a-b}\right.$$

$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) - \frac{2}{15}b^{5/2}e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{2}{15}b^{5/2}e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{4}{15}b^2x \sinh\left(a+\frac{b}{x^2}\right.$$

Mathematica [A] time = 0.110222, size = 102, normalized size = 0.98

$$\frac{1}{15}\left(2\sqrt{\pi}b^{5/2}(\sinh(a)-\cosh(a))\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-2\sqrt{\pi}b^{5/2}(\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+4b^2x\sinh\left(a+\frac{b}{x^2}\right)+3x^5\sinh\left(a+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x^3*Cosh[a + b/x^2] + 2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x]*(-Cosh[a] + Si
nh[a]) - 2*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) + 4*b^2*x*S
inh[a + b/x^2] + 3*x^5*Sinh[a + b/x^2])/15
```

Maple [A] time = 0.059, size = 138, normalized size = 1.3

$$-\frac{e^{-a}x^5}{10}e^{-\frac{b}{x^2}}+\frac{e^{-a}bx^3}{15}e^{-\frac{b}{x^2}}-\frac{2\,e^{-a}\sqrt{\pi}}{15}b^{\frac{5}{2}}\text{Erf}\left(\frac{1}{x}\sqrt{b}\right)-\frac{2\,e^{-a}b^2x}{15}e^{-\frac{b}{x^2}}+\frac{e^{a}x^5}{10}e^{\frac{b}{x^2}}+\frac{e^{a}bx^3}{15}e^{\frac{b}{x^2}}+\frac{2\,e^{a}b^2x}{15}e^{\frac{b}{x^2}}-\frac{2\,e^{a}b^3\sqrt{\pi}}{15}\text{Erf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sinh(a+b/x^2),x)
```

```
[Out] -1/10*exp(-a)*x^5*exp(-b/x^2)+1/15*exp(-a)*b*x^3*exp(-b/x^2)-2/15*exp(-a)*b
^(5/2)*erf(b^(1/2)/x)*Pi^(1/2)-2/15*exp(-a)*exp(-b/x^2)*b^2*x+1/10*exp(a)*x
^5*exp(b/x^2)+1/15*exp(a)*b*exp(b/x^2)*x^3+2/15*exp(a)*b^2*exp(b/x^2)*x-2/1
5*exp(a)*b^3*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A] time = 1.24816, size = 84, normalized size = 0.81

$$\frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{10}\left(x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}e^{(-a)}\Gamma\left(-\frac{3}{2},\frac{b}{x^2}\right)+x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}e^{a}\Gamma\left(-\frac{3}{2},-\frac{b}{x^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*sinh(a + b/x^2) + 1/10*(x^3*(b/x^2)^(3/2)*e^(-a)*gamma(-3/2, b/x^2)
 + x^3*(-b/x^2)^(3/2)*e^a*gamma(-3/2, -b/x^2))*b
```

Fricas [B] time = 1.80541, size = 795, normalized size = 7.64

$$-\frac{3\,x^5-2\,bx^3+4\,b^2x-\left(3\,x^5+2\,bx^3+4\,b^2x\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2-4\,\sqrt{\pi}\left(b^2\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right)+b^2\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/30*(3*x^5 - 2*b*x^3 + 4*b^2*x - (3*x^5 + 2*b*x^3 + 4*b^2*x)*cosh((a*x^2
+ b)/x^2)^2 - 4*sqrt(pi)*(b^2*cosh(a)*cosh((a*x^2 + b)/x^2) + b^2*cosh((a*x
^2 + b)/x^2)*sinh(a) + (b^2*cosh(a) + b^2*sinh(a))*sinh((a*x^2 + b)/x^2))*s
qrt(-b)*erf(sqrt(-b)/x) + 4*sqrt(pi)*(b^2*cosh(a)*cosh((a*x^2 + b)/x^2) - b
^2*cosh((a*x^2 + b)/x^2)*sinh(a) + (b^2*cosh(a) - b^2*sinh(a))*sinh((a*x^2
+ b)/x^2))*sqrt(b)*erf(sqrt(b)/x) - 2*(3*x^5 + 2*b*x^3 + 4*b^2*x)*cosh((a*x
^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) - (3*x^5 + 2*b*x^3 + 4*b^2*x)*sinh((a*x^
2 + b)/x^2)^2)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**4*sinh(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*sinh(a + b/x^2), x)
```

3.42 $\int x^3 \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=62

$$-\frac{1}{4}b^2\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)$$

```
[Out] (b*x^2*Cosh[a + b/x^2])/4 - (b^2*CoshIntegral[b/x^2]*Sinh[a])/4 + (x^4*Sinh
[a + b/x^2])/4 - (b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

Rubi [A] time = 0.113255, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{4}b^2\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b/x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b/x^2])/4 - (b^2*CoshIntegral[b/x^2]*Sinh[a])/4 + (x^4*Sinh
[a + b/x^2])/4 - (b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sinh\left(a+\frac{b}{x^2}\right) dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}\left(b^2\cosh(a)\right)\operatorname{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x^2}\right) - \frac{1}{4}\Big(\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\mathbf{Chi}\left(\frac{b}{x^2}\right)\sinh(a) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\cosh(a)\mathbf{Shi}\left(\frac{b}{x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0415229, size = 56, normalized size = 0.9

$$\frac{1}{4}\left(-b^2\sinh(a)\mathbf{Chi}\left(\frac{b}{x^2}\right) - b^2\cosh(a)\mathbf{Shi}\left(\frac{b}{x^2}\right) + x^4\sinh\left(a+\frac{b}{x^2}\right) + bx^2\cosh\left(a+\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b/x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b/x^2] - b^2*CoshIntegral[b/x^2]*Sinh[a] + x^4*Sinh[a + b/x
^2] - b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

Maple [A] time = 0.031, size = 93, normalized size = 1.5

$$-\frac{e^{-a}x^4}{8}e^{-\frac{b}{x^2}}+\frac{e^{-a}bx^2}{8}e^{-\frac{b}{x^2}}-\frac{e^{-a}b^2}{8}\operatorname{Ei}\left(1,\frac{b}{x^2}\right)+\frac{e^{a}x^4}{8}e^{\frac{b}{x^2}}+\frac{e^{a}bx^2}{8}e^{\frac{b}{x^2}}+\frac{e^{a}b^2}{8}\operatorname{Ei}\left(1,-\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a+b/x^2),x)
```

```
[Out] -1/8*exp(-a)*x^4*exp(-b/x^2)+1/8*exp(-a)*b*x^2*exp(-b/x^2)-1/8*exp(-a)*b^2*
Ei(1,b/x^2)+1/8*exp(a)*x^4*exp(b/x^2)+1/8*exp(a)*b*exp(b/x^2)*x^2+1/8*exp(a
)*b^2*Ei(1,-b/x^2)
```

Maxima [A] time = 1.24101, size = 59, normalized size = 0.95

$$\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{8}\left(be^{(-a)}\Gamma\left(-1,\frac{b}{x^2}\right)-be^{a}\Gamma\left(-1,-\frac{b}{x^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*sinh(a + b/x^2) + 1/8*(b*e^(-a)*gamma(-1, b/x^2) - b*e^a*gamma(-1,
-b/x^2))*b
```

Fricas [A] time = 1.77274, size = 215, normalized size = 3.47

$$\frac{1}{4}x^4\sinh\left(\frac{ax^2+b}{x^2}\right)+\frac{1}{4}bx^2\cosh\left(\frac{ax^2+b}{x^2}\right)-\frac{1}{8}\left(b^2\operatorname{Ei}\left(\frac{b}{x^2}\right)-b^2\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a)-\frac{1}{8}\left(b^2\operatorname{Ei}\left(\frac{b}{x^2}\right)+b^2\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*sinh((a*x^2 + b)/x^2) + 1/4*b*x^2*cosh((a*x^2 + b)/x^2) - 1/8*(b^2*
Ei(b/x^2) - b^2*Ei(-b/x^2))*cosh(a) - 1/8*(b^2*Ei(b/x^2) + b^2*Ei(-b/x^2))*
sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**3*sinh(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(a + b/x^2), x)
```

3.43 $\int x^2 \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=86

$$\frac{1}{3}\sqrt{\pi}e^{-a}b^{3/2}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{3}\sqrt{\pi}e^{a}b^{3/2}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right)$$

```
[Out] (2*b*x*Cosh[a + b/x^2])/3 + (b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(3*E^a) - (b^
(3/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/3 + (x^3*Sinh[a + b/x^2])/3
```

Rubi [A] time = 0.0656322, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5346, 5326, 5327, 5298, 2204, 2205}

$$\frac{1}{3}\sqrt{\pi}e^{-a}b^{3/2}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{3}\sqrt{\pi}e^{a}b^{3/2}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x*Cosh[a + b/x^2])/3 + (b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(3*E^a) - (b^
(3/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/3 + (x^3*Sinh[a + b/x^2])/3
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5327

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*
x)^(m + 1)*Cosh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
```

```
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh\left(a+\frac{b}{x^2}\right) dx &= -\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^4} dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{\cosh\left(a+bx^2\right)}{x^2} dx, x, \frac{1}{x}\right)\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{3}\left(4b^2\right)\text{Subst}\left(\int \sinh\left(a+bx^2\right) dx, x, \frac{1}{x}\right)\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}\left(2b^2\right)\text{Subst}\left(\int e^{-a-bx^2} dx, x, \frac{1}{x}\right) - \frac{1}{3}\left(2b^2\right)\text{Subst}\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}b^{3/2}e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{3}b^{3/2}e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.087626, size = 84, normalized size = 0.98

$$\frac{1}{3}\left(\sqrt{\pi}b^{3/2}(\cosh(a)-\sinh(a))\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\sqrt{\pi}b^{3/2}(\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+x^3\sinh\left(a+\frac{b}{x^2}\right)+2bx\cosh\left(a+\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x*Cosh[a + b/x^2] + b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]
) - b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) + x^3*Sinh[a + b/x
^2])/3
```

Maple [A] time = 0.036, size = 103, normalized size = 1.2

$$-\frac{e^{-a}x^3}{6}e^{-\frac{b}{x^2}}+\frac{e^{-a}\sqrt{\pi}}{3}\mathrm{Erf}\left(\frac{1}{x}\sqrt{b}\right)b^{\frac{3}{2}}+\frac{e^{-a}bx}{3}e^{-\frac{b}{x^2}}+\frac{e^{a}x^3}{6}e^{\frac{b}{x^2}}+\frac{e^{a}bx}{3}e^{\frac{b}{x^2}}-\frac{e^{a}b^2\sqrt{\pi}}{3}\mathrm{Erf}\left(\frac{1}{x}\sqrt{-b}\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b/x^2),x)
```

```
[Out] -1/6*exp(-a)*x^3*exp(-b/x^2)+1/3*exp(-a)*Pi^(1/2)*erf(b^(1/2)/x)*b^(3/2)+1/
3*exp(-a)*exp(-b/x^2)*b*x+1/6*exp(a)*exp(b/x^2)*x^3+1/3*exp(a)*b*exp(b/x^2)
*x-1/3*exp(a)*b^2*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A] time = 1.17987, size = 78, normalized size = 0.91

$$\frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{6}\left(x\sqrt{\frac{b}{x^2}}e^{(-a)}\Gamma\left(-\frac{1}{2},\frac{b}{x^2}\right)+x\sqrt{-\frac{b}{x^2}}e^{a}\Gamma\left(-\frac{1}{2},-\frac{b}{x^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(a + b/x^2) + 1/6*(x*sqrt(b/x^2)*e^(-a)*gamma(-1/2, b/x^2) + x*
sqrt(-b/x^2)*e^a*gamma(-1/2, -b/x^2))*b
```

Fricas [B] time = 1.78067, size = 697, normalized size = 8.1

$$-\frac{x^3-\left(x^3+2\,bx\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2-2\,\sqrt{\pi}\left(b\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right)+b\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right)+\left(b\cosh\left(a\right)+b\sinh\left(a\right)\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3 - (x^3 + 2*b*x)*cosh((a*x^2 + b)/x^2)^2 - 2*sqrt(pi)*(b*cosh(a)*c
osh((a*x^2 + b)/x^2) + b*cosh((a*x^2 + b)/x^2)*sinh(a) + (b*cosh(a) + b*sin
h(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) - 2*sqrt(pi)*(b*cosh(
a)*cosh((a*x^2 + b)/x^2) - b*cosh((a*x^2 + b)/x^2)*sinh(a) + (b*cosh(a) - b
*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) - 2*(x^3 + 2*b*x)*c
osh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) - (x^3 + 2*b*x)*sinh((a*x^2 + b)
/x^2)^2 - 2*b*x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**2*sinh(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(a + b/x^2), x)
```

3.44 $\int x \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=42

$$-\frac{1}{2}b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x^2])/2 + (x^2*Sinh[a + b/x^2])/2 - (b*Sinh[a]*S
inhIntegral[b/x^2])/2
```

Rubi [A] time = 0.0821355, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b/x^2],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x^2])/2 + (x^2*Sinh[a + b/x^2])/2 - (b*Sinh[a]*S
inhIntegral[b/x^2])/2
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sinh\left(a+\frac{b}{x^2}\right)dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\frac{\sinh(a+bx)}{x^2}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{2}b\operatorname{Subst}\left(\int\frac{\cosh(a+bx)}{x}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{2}(b\cosh(a))\operatorname{Subst}\left(\int\frac{\cosh(bx)}{x}\,dx,x,\frac{1}{x^2}\right)-\frac{1}{2}(b\sinh(a))\operatorname{Subst}\left(\int\frac{\sinh}{x}\right.\\
&= -\frac{1}{2}b\cosh(a)\mathbf{Chi}\left(\frac{b}{x^2}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{2}b\sinh(a)\mathbf{Shi}\left(\frac{b}{x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0254909, size = 39, normalized size = 0.93

$$\frac{1}{2}\left(-b\cosh(a)\mathbf{Chi}\left(\frac{b}{x^2}\right)-b\sinh(a)\mathbf{Shi}\left(\frac{b}{x^2}\right)+x^2\sinh\left(a+\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b/x^2],x]
```

```
[Out] (-(b*Cosh[a]*CoshIntegral[b/x^2]) + x^2*Sinh[a + b/x^2] - b*Sinh[a]*SinhInt
egral[b/x^2])/2
```

Maple [A] time = 0.023, size = 58, normalized size = 1.4

$$-\frac{e^{-a}x^2}{4}e^{-\frac{b}{x^2}}+\frac{e^{-a}b}{4}\mathrm{Ei}\left(1,\frac{b}{x^2}\right)+\frac{e^{a}x^2}{4}e^{\frac{b}{x^2}}+\frac{e^{a}b}{4}\mathrm{Ei}\left(1,-\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b/x^2),x)
```

```
[Out] -1/4*exp(-a)*x^2*exp(-b/x^2)+1/4*exp(-a)*b*Ei(1,b/x^2)+1/4*exp(a)*exp(b/x^2
)*x^2+1/4*exp(a)*b*Ei(1,-b/x^2)
```

Maxima [A] time = 1.21857, size = 53, normalized size = 1.26

$$\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}\left(\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)}+\mathrm{Ei}\left(\frac{b}{x^2}\right)e^{a}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh(a + b/x^2) - 1/4*(Ei(-b/x^2)*e^(-a) + Ei(b/x^2)*e^a)*b
```

Fricas [A] time = 1.71276, size = 158, normalized size = 3.76

$$\frac{1}{2}x^2\sinh\left(\frac{ax^2+b}{x^2}\right)-\frac{1}{4}\left(b\mathrm{Ei}\left(\frac{b}{x^2}\right)+b\mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a)-\frac{1}{4}\left(b\mathrm{Ei}\left(\frac{b}{x^2}\right)-b\mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*sinh((a*x^2 + b)/x^2) - 1/4*(b*Ei(b/x^2) + b*Ei(-b/x^2))*cosh(a) -
1/4*(b*Ei(b/x^2) - b*Ei(-b/x^2))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x**2),x)
```

```
[Out] Integral(x*sinh(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(a + b/x^2), x)
```

3.45 $\int \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{\pi}e^{a}\sqrt{b}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] -(Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x])/(2*E^a) - (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqr
t[b]/x])/2 + x*Sinh[a + b/x^2]
```

Rubi [A] time = 0.0436817, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5302, 5326, 5299, 2204, 2205}

$$-\frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{\pi}e^{a}\sqrt{b}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2],x]
```

```
[Out] -(Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x])/(2*E^a) - (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqr
t[b]/x])/2 + x*Sinh[a + b/x^2]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sinh\left(a+\frac{b}{x^2}\right)dx &= -\operatorname{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= x\sinh\left(a+\frac{b}{x^2}\right)-(2b)\operatorname{Subst}\left(\int \cosh\left(a+bx^2\right)dx, x, \frac{1}{x}\right)\\ &= x\sinh\left(a+\frac{b}{x^2}\right)-b\operatorname{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)-b\operatorname{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0669879, size = 70, normalized size = 1.04

$$-\frac{1}{2}\sqrt{\pi}\sqrt{b}\left((\cosh(a)-\sinh(a))\text{Erf}\left(\frac{\sqrt{b}}{x}\right)+(\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)\right)+x\sinh(a)\cosh\left(\frac{b}{x^2}\right)+x\cosh(a)\sinh\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2],x]
```

```
[Out] x*Cosh[b/x^2]*Sinh[a] - (Sqrt[b]*Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a
]) + Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a])))/2 + x*Cosh[a]*Sinh[b/x^2]
```

Maple [A] time = 0.031, size = 70, normalized size = 1.

$$-\frac{\mathrm{e}^{-a}\sqrt{\pi}}{2}\sqrt{b}\,\mathrm{Erf}\left(\frac{1}{x}\sqrt{b}\right)-\frac{\mathrm{e}^{-a}x}{2}\mathrm{e}^{-\frac{b}{x^2}}+\frac{\mathrm{e}^{a}x}{2}\mathrm{e}^{\frac{b}{x^2}}-\frac{\mathrm{e}^{a}b\sqrt{\pi}}{2}\mathrm{Erf}\left(\frac{1}{x}\sqrt{-b}\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2),x)
```

```
[Out] -1/2*exp(-a)*b^(1/2)*Pi^(1/2)*erf(b^(1/2)/x)-1/2*exp(-a)*exp(-b/x^2)*x+1/2*
exp(a)*exp(b/x^2)*x-1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A] time = 1.2211, size = 96, normalized size = 1.43

$$-\frac{1}{2}\,b\left(\frac{\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{\frac{b}{x^2}}\right)-1\right)e^{(-a)}}{x\sqrt{\frac{b}{x^2}}}+\frac{\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-\frac{b}{x^2}}\right)-1\right)e^{a}}{x\sqrt{-\frac{b}{x^2}}}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(pi)*(erf(sqrt(b/x^2)) - 1)*e^(-a)/(x*sqrt(b/x^2)) + sqrt(pi)*(
erf(sqrt(-b/x^2)) - 1)*e^a/(x*sqrt(-b/x^2))) + x*sinh(a + b/x^2)
```

Fricas [B] time = 1.78681, size = 606, normalized size = 9.04

$$\frac{x\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(\cosh(a)+\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{-b}\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) +
cosh((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) + sinh(a))*sinh((a*x^2 + b)/x^2))*
sqrt(-b)*erf(sqrt(-b)/x) - sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) - cosh((
```

```
a*x^2 + b)/x^2)*sinh(a) + (cosh(a) - sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b
)*erf(sqrt(b)/x) + 2*x*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + x*sinh
((a*x^2 + b)/x^2)^2 - x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2),x)
```

```
[Out] Integral(sinh(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2), x)
```

3.46 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

```
[Out] -(CoshIntegral[b/x^2]*Sinh[a])/2 - (Cosh[a]*SinhIntegral[b/x^2])/2
```

Rubi [A] time = 0.0329483, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5318, 5317, 5316}

$$-\frac{1}{2}\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x,x]
```

```
[Out] -(CoshIntegral[b/x^2]*Sinh[a])/2 - (Cosh[a]*SinhIntegral[b/x^2])/2
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \cosh(a)\int \frac{\sinh\left(\frac{b}{x^2}\right)}{x}\,dx + \sinh(a)\int \frac{\cosh\left(\frac{b}{x^2}\right)}{x}\,dx$$
$$= -\frac{1}{2}\mathbf{Chi}\left(\frac{b}{x^2}\right)\sinh(a) - \frac{1}{2}\cosh(a)\mathbf{Shi}\left(\frac{b}{x^2}\right)$$

Mathematica [A] time = 0.0139663, size = 25, normalized size = 1.

$$\frac{1}{2}\left(\sinh(a)\left(-\mathbf{Chi}\left(\frac{b}{x^2}\right)\right) - \cosh(a)\mathbf{Shi}\left(\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x,x]
```

```
[Out] (-(CoshIntegral[b/x^2]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x^2])/2
```

Maple [A] time = 0.023, size = 27, normalized size = 1.1

$$-\frac{e^{-a}}{4}\mathrm{Ei}\left(1,\frac{b}{x^2}\right) + \frac{e^{a}}{4}\mathrm{Ei}\left(1,-\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x,x)
```

```
[Out] -1/4*exp(-a)*Ei(1,b/x^2)+1/4*exp(a)*Ei(1,-b/x^2)
```

Maxima [A] time = 1.38342, size = 32, normalized size = 1.28

$$\frac{1}{4}\,\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)} - \frac{1}{4}\,\mathrm{Ei}\left(\frac{b}{x^2}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(-b/x^2)*e^(-a) - 1/4*Ei(b/x^2)*e^a
```

Fricas [A] time = 1.70199, size = 105, normalized size = 4.2

$$-\frac{1}{4}\left(\mathrm{Ei}\left(\frac{b}{x^2}\right)-\mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh\left(a\right)-\frac{1}{4}\left(\mathrm{Ei}\left(\frac{b}{x^2}\right)+\mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(Ei(b/x^2) - Ei(-b/x^2))*cosh(a) - 1/4*(Ei(b/x^2) + Ei(-b/x^2))*sinh(a
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x, x)
```

3.47 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]/x])/(4*Sqrt[b]*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/
(4*Sqrt[b])
```

Rubi [A] time = 0.0311093, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5346, 5298, 2204, 2205}

$$\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^2,x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]/x])/(4*Sqrt[b]*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/
(4*Sqrt[b])
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)-\frac{1}{2}\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{e^{-a}\sqrt{\pi}\,\mathbf{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{e^{a}\sqrt{\pi}\,\mathbf{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0343331, size = 50, normalized size = 0.88

$$\frac{\sqrt{\pi}\left((\cosh(a)-\sinh(a))\mathbf{Erf}\left(\frac{\sqrt{b}}{x}\right)-(\sinh(a)+\cosh(a))\mathbf{Erfi}\left(\frac{\sqrt{b}}{x}\right)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^2,x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) - Erfi[Sqrt[b]/x]*(Cosh[a] +
Sinh[a])))/(4*Sqrt[b])
```

Maple [A] time = 0.027, size = 44, normalized size = 0.8

$$\frac{\sqrt{\pi}\mathrm{e}^{-a}}{4}\mathrm{Erf}\left(\frac{1}{x}\sqrt{b}\right)\frac{1}{\sqrt{b}}-\frac{\mathrm{e}^{a}\sqrt{\pi}}{4}\mathrm{Erf}\left(\frac{1}{x}\sqrt{-b}\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^2,x)
```

```
[Out] 1/4*erf(b^(1/2)/x)*Pi^(1/2)*exp(-a)/b^(1/2)-1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*
erf((-b)^(1/2)/x)
```

Maxima [A] time = 1.17297, size = 84, normalized size = 1.47

$$-\frac{1}{2}b\left(\frac{e^{(-a)}\Gamma\left(\frac{3}{2},\frac{b}{x^2}\right)}{x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}+\frac{e^{a}\Gamma\left(\frac{3}{2},-\frac{b}{x^2}\right)}{x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-a)*gamma(3/2, b/x^2)/(x^3*(b/x^2)^(3/2)) + e^a*gamma(3/2, -b/x^
2)/(x^3*(-b/x^2)^(3/2))) - sinh(a + b/x^2)/x
```

Fricas [A] time = 1.82332, size = 158, normalized size = 2.77

$$\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)+\sqrt{\pi}\sqrt{b}(\cosh(a)-\sinh(a))\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)/x) + sqrt(pi)*sqrt(
b)*(cosh(a) - sinh(a))*erf(sqrt(b)/x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**2,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + \frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^2, x)
```

3.48 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3} dx$

Optimal. Leaf size=15

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -Cosh[a + b/x^2]/(2*b)
```

Rubi [A] time = 0.0194359, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^3,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

Mathematica [A] time = 0.0046271, size = 15, normalized size = 1.

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^3,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$-\frac{1}{2\,b}\cosh\left(a+\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^3,x)
```

```
[Out] -1/2*cosh(a+b/x^2)/b
```

Maxima [A] time = 1.1396, size = 18, normalized size = 1.2

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*cosh(a + b/x^2)/b
```

Fricas [A] time = 1.61495, size = 41, normalized size = 2.73

$$-\frac{\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*cosh((a*x^2 + b)/x^2)/b
```

Sympy [A] time = 4.68851, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b} & \text{for}\ b \neq 0 \\ -\frac{\sinh\left(a\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**3,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b), Ne(b, 0)), (-sinh(a)/(2*x**2), True))
```

Giac [A] time = 1.2381, size = 34, normalized size = 2.27

$$-\frac{e^{\left(a+\frac{b}{x^2}\right)}+e^{\left(-a-\frac{b}{x^2}\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(e^(a + b/x^2) + e^(-a - b/x^2))/b
```

3.49 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[Out] -Cosh[a + b/x^2]/(2*b*x) + (Sqrt[Pi]*Erf[Sqrt[b]/x])/(8*b^(3/2)*E^a) + (E^a
*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(8*b^(3/2))
```

Rubi [A] time = 0.0480669, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5346, 5324, 5299, 2204, 2205}

$$\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^4,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b*x) + (Sqrt[Pi]*Erf[Sqrt[b]/x])/(8*b^(3/2)*E^a) + (E^a
*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(8*b^(3/2))
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\text{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)}{4b} + \frac{\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)}{4b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0710922, size = 74, normalized size = 0.99

$$\frac{\sqrt{\pi}x(\cosh(a)-\sinh(a))\text{Erf}\left(\frac{\sqrt{b}}{x}\right)+\sqrt{\pi}x(\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)-4\sqrt{b}\cosh\left(a+\frac{b}{x^2}\right)}{8b^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^4,x]
```

```
[Out] (-4*Sqrt[b]*Cosh[a + b/x^2] + Sqrt[Pi]*x*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a])
 + Sqrt[Pi]*x*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]))/(8*b^(3/2)*x)
```

Maple [A] time = 0.035, size = 82, normalized size = 1.1

$$-\frac{e^{-a}}{4\,bx}e^{-\frac{b}{x^2}} + \frac{e^{-a}\sqrt{\pi}}{8}\mathrm{Erf}\left(\frac{1}{x}\sqrt{b}\right)b^{-\frac{3}{2}} - \frac{e^{a}}{4\,bx}e^{\frac{b}{x^2}} + \frac{e^{a}\sqrt{\pi}}{8\,b}\mathrm{Erf}\left(\frac{1}{x}\sqrt{-b}\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^4,x)
```

```
[Out] -1/4*exp(-a)/b/x*exp(-b/x^2)+1/8*exp(-a)/b^(3/2)*Pi^(1/2)*erf(b^(1/2)/x)-1/
4*exp(a)*exp(b/x^2)/x/b+1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A] time = 1.26549, size = 84, normalized size = 1.12

$$-\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{5}{2},\frac{b}{x^2}\right)}{x^5\left(\frac{b}{x^2}\right)^{\frac{5}{2}}} + \frac{e^{a}\Gamma\left(\frac{5}{2},-\frac{b}{x^2}\right)}{x^5\left(-\frac{b}{x^2}\right)^{\frac{5}{2}}}\right) - \frac{\sinh\left(a+\frac{b}{x^2}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(e^(-a)*gamma(5/2, b/x^2)/(x^5*(b/x^2)^(5/2)) + e^a*gamma(5/2, -b/x^
2)/(x^5*(-b/x^2)^(5/2))) - 1/3*sinh(a + b/x^2)/x^3
```

Fricas [B] time = 1.77908, size = 653, normalized size = 8.71

$$-\frac{2\,b\cosh\left(\frac{ax^2+b}{x^2}\right)^2 + \sqrt{\pi}\left(x\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right) + x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right) + \left(x\cosh\left(a\right) + x\sinh\left(a\right)\right)\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{-b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x^
2) + x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) + x*sinh(a))*sinh((a*x^2
+ b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) - sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/
x^2) - x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) - x*sinh(a))*sinh((a*x^
2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) + 4*b*cosh((a*x^2 + b)/x^2)*sinh((a*x^2
 + b)/x^2) + 2*b*sinh((a*x^2 + b)/x^2)^2 + 2*b)/(b^2*x*cosh((a*x^2 + b)/x^2
) + b^2*x*sinh((a*x^2 + b)/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**4,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^4, x)
```

3.50 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5} dx$

Optimal. Leaf size=34

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}$$

```
[Out] -Cosh[a + b/x^2]/(2*b*x^2) + Sinh[a + b/x^2]/(2*b^2)
```

Rubi [A] time = 0.0335038, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 3296, 2637}

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^5,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b*x^2) + Sinh[a + b/x^2]/(2*b^2)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2} + \frac{\operatorname{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{2b}$$
$$= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}$$

Mathematica [A] time = 0.0273929, size = 34, normalized size = 1.

$$\frac{x^2\sinh\left(a+\frac{b}{x^2}\right) - b\cosh\left(a+\frac{b}{x^2}\right)}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^5,x]
```

```
[Out] (-(b*Cosh[a + b/x^2]) + x^2*Sinh[a + b/x^2])/(2*b^2*x^2)
```

Maple [A] time = 0.025, size = 55, normalized size = 1.6

$$-\frac{-x^2+b}{4\,x^2b^2}\mathrm{e}^{\frac{ax^2+b}{x^2}} - \frac{x^2+b}{4\,x^2b^2}\mathrm{e}^{-\frac{ax^2+b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^5,x)
```

```
[Out] -1/4*(-x^2+b)/x^2/b^2*exp((a*x^2+b)/x^2)-1/4*(x^2+b)/x^2/b^2*exp(-(a*x^2+b)
/x^2)
```

Maxima [C] time = 1.31073, size = 65, normalized size = 1.91

$$-\frac{1}{8}b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x^2}\right)}{b^3} - \frac{e^{a}\Gamma\left(3,-\frac{b}{x^2}\right)}{b^3}\right) - \frac{\sinh\left(a+\frac{b}{x^2}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*(e^(-a)*gamma(3, b/x^2)/b^3 - e^a*gamma(3, -b/x^2)/b^3) - 1/4*sinh(a
 + b/x^2)/x^4
```

Fricas [A] time = 1.71071, size = 93, normalized size = 2.74

$$\frac{x^2\sinh\left(\frac{ax^2+b}{x^2}\right)-b\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*sinh((a*x^2 + b)/x^2) - b*cosh((a*x^2 + b)/x^2))/(b^2*x^2)
```

Sympy [A] time = 15.0673, size = 37, normalized size = 1.09

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}+\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2} & \text{for } b\neq 0\\ -\frac{\sinh(a)}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**5,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b*x**2) + sinh(a + b/x**2)/(2*b**2), Ne(b,
0)), (-sinh(a)/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^5, x)
```

3.51 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$

Optimal. Leaf size=93

$$\frac{3\sqrt{\pi}e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}-\frac{3\sqrt{\pi}e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}$$

```
[Out] -Cosh[a + b/x^2]/(2*b*x^3) + (3*Sqrt[Pi]*Erf[Sqrt[b]/x])/(16*b^(5/2)*E^a) -
 (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(16*b^(5/2)) + (3*Sinh[a + b/x^2])/(4*b^2
*x)
```

Rubi [A] time = 0.0682972, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5346, 5324, 5325, 5298, 2204, 2205}

$$\frac{3\sqrt{\pi}e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}-\frac{3\sqrt{\pi}e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^6,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b*x^3) + (3*Sqrt[Pi]*Erf[Sqrt[b]/x])/(16*b^(5/2)*E^a) -
 (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(16*b^(5/2)) + (3*Sinh[a + b/x^2])/(4*b^2
*x)
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx &= -\text{Subst}\left(\int x^4\sinh\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3\,\text{Subst}\left(\int x^2\cosh\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}-\frac{3\,\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)}{4b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}+\frac{3\,\text{Subst}\left(\int e^{-a-bx^2}\,dx,x,\frac{1}{x}\right)}{8b^2}-\frac{3\,\text{Subst}\left(\int e^{a+bx^2}\,dx,x,\frac{1}{x}\right)}{8b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}-\frac{3e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}
\end{aligned}
$$

Mathematica [A] time = 0.124699, size = 97, normalized size = 1.04

$$\frac{3\sqrt{\pi}x^3(\cosh(a)-\sinh(a))\mathbf{Erf}\left(\frac{\sqrt{b}}{x}\right)-3\sqrt{\pi}x^3(\sinh(a)+\cosh(a))\mathbf{Erfi}\left(\frac{\sqrt{b}}{x}\right)+4\sqrt{b}\left(3x^2\sinh\left(a+\frac{b}{x^2}\right)-2b\cosh\left(a+\frac{b}{x^2}\right)\right)}{16b^{5/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^6,x]
```

```
[Out] (3*Sqrt[Pi]*x^3*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) - 3*Sqrt[Pi]*x^3*Erfi[Sq
rt[b]/x]*(Cosh[a] + Sinh[a]) + 4*Sqrt[b]*(-2*b*Cosh[a + b/x^2] + 3*x^2*Sinh
[a + b/x^2]))/(16*b^(5/2)*x^3)
```

Maple [A] time = 0.043, size = 117, normalized size = 1.3

$$-\frac{e^{-a}}{4bx^3}e^{-\frac{b}{x^2}}-\frac{3e^{-a}}{8b^2x}e^{-\frac{b}{x^2}}+\frac{3e^{-a}\sqrt{\pi}}{16}\mathrm{Erf}\left(\frac{1}{x}\sqrt{b}\right)b^{-\frac{5}{2}}-\frac{e^{a}}{4bx^3}e^{\frac{b}{x^2}}+\frac{3e^{a}}{8b^2x}e^{\frac{b}{x^2}}-\frac{3e^{a}\sqrt{\pi}}{16b^2}\mathrm{Erf}\left(\frac{1}{x}\sqrt{-b}\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^6,x)
```

```
[Out] -1/4*exp(-a)/b/x^3*exp(-b/x^2)-3/8*exp(-a)/b^2/x*exp(-b/x^2)+3/16*exp(-a)/b
^(5/2)*Pi^(1/2)*erf(b^(1/2)/x)-1/4*exp(a)*exp(b/x^2)/x^3/b+3/8*exp(a)/b^2*e
xp(b/x^2)/x-3/16*exp(a)/b^2*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A] time = 1.27342, size = 84, normalized size = 0.9

$$-\frac{1}{10}b\left(\frac{e^{(-a)}\Gamma\left(\frac{7}{2},\frac{b}{x^2}\right)}{x^7\left(\frac{b}{x^2}\right)^{\frac{7}{2}}}+\frac{e^{a}\Gamma\left(\frac{7}{2},-\frac{b}{x^2}\right)}{x^7\left(-\frac{b}{x^2}\right)^{\frac{7}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*b*(e^(-a)*gamma(7/2, b/x^2)/(x^7*(b/x^2)^(7/2)) + e^a*gamma(7/2, -b/x
^2)/(x^7*(-b/x^2)^(7/2))) - 1/5*sinh(a + b/x^2)/x^5
```

Fricas [B] time = 1.86788, size = 768, normalized size = 8.26

$$-\frac{6\,bx^2-2\left(3\,bx^2-2\,b^2\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2-3\,\sqrt{\pi}\left(x^3\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right)+x^3\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right)+\left(x^3\cosh\left(a\right)+x^3\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x^2 - 2*(3*b*x^2 - 2*b^2)*cosh((a*x^2 + b)/x^2)^2 - 3*sqrt(pi)*(
x^3*cosh(a)*cosh((a*x^2 + b)/x^2) + x^3*cosh((a*x^2 + b)/x^2)*sinh(a) + (x^
3*cosh(a) + x^3*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) -
3*sqrt(pi)*(x^3*cosh(a)*cosh((a*x^2 + b)/x^2) - x^3*cosh((a*x^2 + b)/x^2)*s
inh(a) + (x^3*cosh(a) - x^3*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqr
t(b)/x) - 4*(3*b*x^2 - 2*b^2)*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) -
 2*(3*b*x^2 - 2*b^2)*sinh((a*x^2 + b)/x^2)^2 + 4*b^2)/(b^3*x^3*cosh((a*x^2
+ b)/x^2) + b^3*x^3*sinh((a*x^2 + b)/x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**6,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^6, x)
```

3.52 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx$

Optimal. Leaf size=47

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}$$

```
[Out] -(Cosh[a + b/x^2]/b^3) - Cosh[a + b/x^2]/(2*b*x^4) + Sinh[a + b/x^2]/(b^2*x
^2)
```

Rubi [A] time = 0.0586295, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5320, 3296, 2638}

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^7,x]
```

```
[Out] -(Cosh[a + b/x^2]/b^3) - Cosh[a + b/x^2]/(2*b*x^4) + Sinh[a + b/x^2]/(b^2*x
^2)
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2} - \frac{\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}
\end{aligned}$$

Mathematica [A] time = 0.043287, size = 44, normalized size = 0.94

$$\frac{2bx^2\sinh\left(a+\frac{b}{x^2}\right) - \left(b^2+2x^4\right)\cosh\left(a+\frac{b}{x^2}\right)}{2b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^7,x]
```

```
[Out] (-((b^2 + 2*x^4)*Cosh[a + b/x^2]) + 2*b*x^2*Sinh[a + b/x^2])/(2*b^3*x^4)
```

Maple [A] time = 0.035, size = 73, normalized size = 1.6

$$-\frac{2\,x^4-2\,bx^2+b^2}{4\,b^3x^4}{{\rm e}^{\frac{ax^2+b}{x^2}}}-\frac{2\,x^4+2\,bx^2+b^2}{4\,b^3x^4}{{\rm e}^{-\frac{ax^2+b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^7,x)
```

```
[Out] -1/4*(2*x^4-2*b*x^2+b^2)/b^3/x^4*exp((a*x^2+b)/x^2)-1/4*(2*x^4+2*b*x^2+b^2)
/b^3/x^4*exp(-(a*x^2+b)/x^2)
```

Maxima [C] time = 1.1839, size = 63, normalized size = 1.34

$$-\frac{1}{12}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x^2}\right)}{b^4}+\frac{e^{a}\Gamma\left(4,-\frac{b}{x^2}\right)}{b^4}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*b*(e^(-a)*gamma(4, b/x^2)/b^4 + e^a*gamma(4, -b/x^2)/b^4) - 1/6*sinh(
a + b/x^2)/x^6
```

Fricas [A] time = 1.65089, size = 115, normalized size = 2.45

$$\frac{2\,bx^2\sinh\left(\frac{ax^2+b}{x^2}\right)-\left(2\,x^4+b^2\right)\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*sinh((a*x^2 + b)/x^2) - (2*x^4 + b^2)*cosh((a*x^2 + b)/x^2))/(
b^3*x^4)
```

Sympy [A] time = 48.2079, size = 51, normalized size = 1.09

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}+\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} & \text{for}\ b\neq 0 \\ -\frac{\sinh(a)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**7,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b*x**4) + sinh(a + b/x**2)/(b**2*x**2) - co
sh(a + b/x**2)/b**3, Ne(b, 0)), (-sinh(a)/(6*x**6), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^7, x)
```

3.53 $\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=194

$$\frac{1}{16}e^{3a}3^{\frac{m+1}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{3b}{x^2}\right)-\frac{3}{16}e^{a}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)+\frac{3}{16}e^{-a}x\left(\right.$$

```
[Out] (3^((1 + m)/2)*E^(3*a)*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
-3*b)/x^2])/16 - (3*E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2,
-(b/x^2)])/16 + (3*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/
(16*E^a) - (3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
3*b)/x^2])/(16*E^(3*a))
```

Rubi [A] time = 0.221037, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5350, 5340, 5328, 2218}

$$\frac{1}{16}e^{3a}3^{\frac{m+1}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{3b}{x^2}\right)-\frac{3}{16}e^{a}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)+\frac{3}{16}e^{-a}x\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x^2]^3,x]
```

```
[Out] (3^((1 + m)/2)*E^(3*a)*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
-3*b)/x^2])/16 - (3*E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2,
-(b/x^2)])/16 + (3*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/
(16*E^a) - (3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
3*b)/x^2])/(16*E^(3*a))
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right) dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh^3\left(a+bx^2\right) dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(-\frac{3}{4}x^{-2-m}\sinh\left(a+bx^2\right)+\frac{1}{4}x^{-2-m}\sinh\left(3a+3bx^2\right)\right) dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\sinh\left(3a+3bx^2\right) dx, x, \frac{1}{x}\right)\right)+\frac{1}{4}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subs}\\
&= \frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-3a-3bx^2}x^{-2-m} dx, x, \frac{1}{x}\right)-\frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{3a+3bx^2}x^{-}\right.\\
&= \frac{1}{16}3^{\frac{1+m}{2}}e^{3a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{3b}{x^2}\right)-\frac{3}{16}e^a\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\right.\end{aligned}$$

Mathematica [B] time = 24.2724, size = 1039, normalized size = 5.36

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2]^3,x]
```

```
[Out] ((e*x)^m*Cosh[a]^3*((-3*(((-(b/x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2
, -(b/x^2)])/2 - ((b/x^2)^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, b/x^2])/2
))/8 + ((3^((1 + m)/2)*(-(b/x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, (
```

```
-3*b)/x^2])/2 - (3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)
/2, (3*b)/x^2])/2)/8))/x^m + (3*x*(e*x)^m*Cosh[a]^2*(-4*Cosh[b/x^2] + 4*Cos
h[(3*b)/x^2] - 3^((1 + m)/2)*m*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, (-3
*b)/x^2] + m*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, -(b/x^2)] + m*(b/x^2)
^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2] - 3^((1 + m)/2)*m*(b/x^2)^((1 + m)/2)
*Gamma[(-1 - m)/2, (3*b)/x^2] - 2*3^((1 + m)/2)*(-(b/x^2))^((1 + m)/2)*Gamm
a[(1 - m)/2, (-3*b)/x^2] + 2*(-(b/x^2))^((1 + m)/2)*Gamma[(1 - m)/2, -(b/x^
2)] + 2*(b/x^2)^((1 + m)/2)*Gamma[(1 - m)/2, b/x^2] - 2*3^((1 + m)/2)*(b/x^
2)^((1 + m)/2)*Gamma[(1 - m)/2, (3*b)/x^2])*Sinh[a])/16 + ((e*x)^m*((3*(((-
(b/x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, -(b/x^2)])/2 + ((b/x^2)^((
1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, b/x^2])/2))/8 + ((3^((1 + m)/2)*(-(b/
x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, (-3*b)/x^2])/2 + (3^((1 + m)/
2)*(b/x^2)^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, (3*b)/x^2])/2)/8)*Sinh[a
]^3)/x^m + (3*x*(e*x)^m*Cosh[a]*Sinh[a]^2*(-(3^((1 + m)/2)*m*(-(b/x^2))^((1
 + m)/2)*Gamma[(-1 - m)/2, (-3*b)/x^2]) - m*(-(b/x^2))^((1 + m)/2)*Gamma[(-
1 - m)/2, -(b/x^2)] + m*(b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2] + 3^((
1 + m)/2)*m*(b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, (3*b)/x^2] - 2*3^((1 + m)
/2)*(-(b/x^2))^((1 + m)/2)*Gamma[(1 - m)/2, (-3*b)/x^2] - 2*(-(b/x^2))^((1
+ m)/2)*Gamma[(1 - m)/2, -(b/x^2)] + 2*(b/x^2)^((1 + m)/2)*Gamma[(1 - m)/2,
 b/x^2] + 2*3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*Gamma[(1 - m)/2, (3*b)/x^2] +
 4*Sinh[b/x^2] + 4*Sinh[(3*b)/x^2]))/16
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh\left(a+\frac{b}{x^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x^2)^3,x)

[Out] int((e*x)^m*sinh(a+b/x^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax^2+b}{x^2}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x**2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^3, x)
```

3.54 $\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=117

$$e^{2a}2^{\frac{m-5}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{2b}{x^2}\right)+e^{-2a}2^{\frac{m-5}{2}}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),\frac{2b}{x^2}\right)-\frac{x(ex)^m}{2(m+1)}$$

```
[Out] -(x*(e*x)^m)/(2*(1 + m)) + 2^((-5 + m)/2)*E^(2*a)*(-(b/x^2))^((1 + m)/2)*x*
(e*x)^m*Gamma[(-1 - m)/2, (-2*b)/x^2] + (2^((-5 + m)/2)*(b/x^2)^((1 + m)/2)
*x*(e*x)^m*Gamma[(-1 - m)/2, (2*b)/x^2])/E^(2*a)
```

Rubi [A] time = 0.165759, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5350, 5340, 5329, 2218}

$$e^{2a}2^{\frac{m-5}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{2b}{x^2}\right)+e^{-2a}2^{\frac{m-5}{2}}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),\frac{2b}{x^2}\right)-\frac{x(ex)^m}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x^2]^2,x]
```

```
[Out] -(x*(e*x)^m)/(2*(1 + m)) + 2^((-5 + m)/2)*E^(2*a)*(-(b/x^2))^((1 + m)/2)*x*
(e*x)^m*Gamma[(-1 - m)/2, (-2*b)/x^2] + (2^((-5 + m)/2)*(b/x^2)^((1 + m)/2)
*x*(e*x)^m*Gamma[(-1 - m)/2, (2*b)/x^2])/E^(2*a)
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5329

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right) dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh^2\left(a+bx^2\right) dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(-\frac{1}{2}x^{-2-m}+\frac{1}{2}x^{-2-m}\cosh\left(2a+2bx^2\right)\right) dx, x, \frac{1}{x}\right)\right) \\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\cosh\left(2a+2bx^2\right) dx, x, \frac{1}{x}\right) \\
&= -\frac{x(ex)^m}{2(1+m)} - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-2a-2bx^2}x^{-2-m} dx, x, \frac{1}{x}\right) - \frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst} \\
&= -\frac{x(ex)^m}{2(1+m)} + 2^{\frac{1}{2}(-5+m)}e^{2a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), -\frac{2b}{x^2}\right) + 2^{\frac{1}{2}(-5+m)}e^{-2a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(e
\end{aligned}$$

Mathematica [A] time = 0.799796, size = 122, normalized size = 1.04

$$\frac{x(ex)^m\left(2^{\frac{m+1}{2}}(m+1)(\sinh(2a)+\cosh(2a))\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{1}{2}(-m-1), -\frac{2b}{x^2}\right)+2^{\frac{m+1}{2}}(m+1)(\cosh(2a)-\sinh(2a))\left(\right.\right.}{8(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2]^2,x]
```

```
[Out] (x*(e*x)^m*(-4 + 2^((1 + m)/2)*(1 + m)*(b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2
, (2*b)/x^2]*(Cosh[2*a] - Sinh[2*a]) + 2^((1 + m)/2)*(1 + m)*(-(b/x^2))^((1
 + m)/2)*Gamma[(-1 - m)/2, (-2*b)/x^2]*(Cosh[2*a] + Sinh[2*a])))/(8*(1 + m)
)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh\left(a+\frac{b}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x^2)^2,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x^2)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax^2+b}{x^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x**2)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + \frac{b}{x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^2, x)
```

3.55 $\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=87

$$\frac{1}{4}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)-\frac{1}{4}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right)$$

```
[Out] (E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, -(b/x^2)])/4 - ((b/
x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/(4*E^a)
```

Rubi [A] time = 0.0869012, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5350, 5328, 2218}

$$\frac{1}{4}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)-\frac{1}{4}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x^2],x]
```

```
[Out] (E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, -(b/x^2)])/4 - ((b/
x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/(4*E^a)
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
```

```
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh\left(a+bx^2\right)dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{-a-bx^2}x^{-2-m}\,dx, x, \frac{1}{x}\right) - \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{a+bx^2}x^{-2-m}$$
$$= \frac{1}{4}e^a\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), -\frac{b}{x^2}\right) - \frac{1}{4}e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m\Gamma\left(\frac{1}{2}(-1-m), \frac{b}{x^2}\right)$$

Mathematica [A] time = 0.139264, size = 84, normalized size = 0.97

$$\frac{1}{4}x(ex)^m\left((\sinh(a)+\cosh(a))\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{1}{2}(-m-1), -\frac{b}{x^2}\right) - (\cosh(a)-\sinh(a))\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{1}{2}(-m-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2],x]
```

```
[Out] (x*(e*x)^m*(-((b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2]*(Cosh[a] - Sinh[
a])) + (-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, -(b/x^2)]*(Cosh[a] + Sinh[a
])))/4
```

Maple [C] time = 0.034, size = 77, normalized size = 0.9

$$\frac{(ex)^m\, b\cosh(a)}{(-1+m)\,x}\,{}_1F_2\left(\frac{1}{4}-\frac{m}{4};\ \frac{3}{2},\frac{5}{4}-\frac{m}{4};\ \frac{b^2}{4\,x^4}\right) + \frac{(ex)^m\, x\sinh(a)}{1+m}\,{}_1F_2\left(-\frac{1}{4}-\frac{m}{4};\ \frac{1}{2},\frac{3}{4}-\frac{m}{4};\ \frac{b^2}{4\,x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x^2),x)
```

```
[Out] (e*x)^m*b/(-1+m)/x*hypergeom([1/4-1/4*m],[3/2,5/4-1/4*m],1/4/x^4*b^2)*cosh(
a)+(e*x)^m/(1+m)*x*hypergeom([-1/4-1/4*m],[1/2,3/4-1/4*m],1/4/x^4*b^2)*sinh
```

(a)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax^2 + b}{x^2}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x**2),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2), x)
```

3.56 $\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=25

$$x^{-m}(ex)^m \text{Unintegrable}\left(x^m \operatorname{csch}\left(a+\frac{b}{x^2}\right),x\right)$$

```
[Out] ((e*x)^m*Unintegrable[x^m*Csch[a + b/x^2], x])/x^m
```

Rubi [A] time = 0.0240249, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b/x^2],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b/x^2], x])/x^m
```

Rubi steps

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right) dx = \left(x^{-m}(ex)^m\right) \int x^m \operatorname{csch}\left(a+\frac{b}{x^2}\right) dx$$

Mathematica [A] time = 3.13314, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{csch}\left(a+\frac{b}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b/x^2],x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b/x^2], x]
```

Maple [A] time = 0.034, size = 0, normalized size = 0.

$$\int (ex)^m \left(\sinh\left(a+\frac{b}{x^2}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a+b/x^2),x)
```

```
[Out] int((e*x)^m/sinh(a+b/x^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/sinh(a + b/x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{\sinh\left(\frac{ax^2+b}{x^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/sinh((a*x^2 + b)/x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(a+b/x**2),x)
```

```
[Out] Integral((e*x)**m/sinh(a + b/x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(a + b/x^2), x)
```

3.57 $\int \frac{\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\cosh\left(\sqrt{x}\right)$$

```
[Out] 2*Cosh[Sqrt[x]]
```

Rubi [A] time = 0.0119936, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$2\cosh\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Cosh[Sqrt[x]]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \sinh(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cosh\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0028366, size = 8, normalized size = 1.

$$2\cosh\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Cosh[Sqrt[x]]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$2\cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*cosh(x^(1/2))
```

Maxima [A] time = 1.04629, size = 8, normalized size = 1.

$$2\cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*cosh(sqrt(x))
```

Fricas [A] time = 1.84414, size = 23, normalized size = 2.88

$$2\cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*cosh(sqrt(x))
```

Sympy [A] time = 0.388617, size = 7, normalized size = 0.88

$$2\cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*cosh(sqrt(x))
```

Giac [A] time = 1.16501, size = 15, normalized size = 1.88

$$e^{\left(-\sqrt{x}\right)} + e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] e^(-sqrt(x)) + e^sqrt(x)
```

3.58 $\int x^2 \sinh(a + bx^n)\, dx$

Optimal. Leaf size=75

$$\frac{e^{-a}x^3 (bx^n)^{-3/n} \operatorname{Gamma}\left(\frac{3}{n}, bx^n\right)}{2n} - \frac{e^a x^3 (-bx^n)^{-3/n} \operatorname{Gamma}\left(\frac{3}{n}, -bx^n\right)}{2n}$$

```
[Out] -(E^a*x^3*Gamma[3/n, -(b*x^n)])/(2*n*(-(b*x^n))^(3/n)) + (x^3*Gamma[3/n, b*
x^n])/(2*E^a*n*(b*x^n)^(3/n))
```

Rubi [A] time = 0.0738682, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5360, 2218}

$$\frac{e^{-a}x^3 (bx^n)^{-3/n} \operatorname{Gamma}\left(\frac{3}{n}, bx^n\right)}{2n} - \frac{e^a x^3 (-bx^n)^{-3/n} \operatorname{Gamma}\left(\frac{3}{n}, -bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*x^3*Gamma[3/n, -(b*x^n)])/(2*n*(-(b*x^n))^(3/n)) + (x^3*Gamma[3/n, b*
x^n])/(2*E^a*n*(b*x^n)^(3/n))
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^2 \sinh(a+bx^n)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}x^2\,dx\right)+\frac{1}{2}\int e^{a+bx^n}x^2\,dx$$
$$= -\frac{e^a x^3 (-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-bx^n\right)}{2n}+\frac{e^{-a}x^3 (bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},bx^n\right)}{2n}$$

Mathematica [A] time = 0.0947994, size = 88, normalized size = 1.17

$$-\frac{x^3\left(-b^2x^{2n}\right)^{-3/n}\left((\sinh(a)+\cosh(a))\,(bx^n)^{3/n}\,\text{Gamma}\left(\frac{3}{n},-bx^n\right)-(\cosh(a)-\sinh(a))\,(-bx^n)^{3/n}\,\text{Gamma}\left(\frac{3}{n},bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^n],x]
```

```
[Out] -(x^3*(-((-(b*x^n))^(3/n)*Gamma[3/n, b*x^n]*(Cosh[a] - Sinh[a])) + (b*x^n)^
(3/n)*Gamma[3/n, -(b*x^n)]*(Cosh[a] + Sinh[a])))/(2*n*(-(b^2*x^(2*n)))^(3/n
))
```

Maple [C] time = 0.09, size = 77, normalized size = 1.

$$\frac{x^3\sinh(a)}{3}\,{}_1\mathrm{F}_2\left(\frac{3}{2n};\frac{1}{2},1+\frac{3}{2n};\frac{x^{2n}b^2}{4}\right)+\frac{x^{n+3}b\cosh(a)}{n+3}\,{}_1\mathrm{F}_2\left(\frac{1}{2}+\frac{3}{2n};\frac{3}{2},\frac{3}{2}+\frac{3}{2n};\frac{x^{2n}b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*x^n),x)
```

```
[Out] 1/3*x^3*hypergeom([3/2/n],[1/2,1+3/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+3)*x^
(n+3)*b*hypergeom([1/2+3/2/n],[3/2,3/2+3/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Maxima [A] time = 1.2102, size = 99, normalized size = 1.32

$$\frac{x^3e^{(-a)}\Gamma\left(\frac{3}{n},bx^n\right)}{2\,(bx^n)^{\frac{3}{n}}\,n}-\frac{x^3e^a\Gamma\left(\frac{3}{n},-bx^n\right)}{2\,(-bx^n)^{\frac{3}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^3*e^(-a)*gamma(3/n, b*x^n)/((b*x^n)^(3/n)*n) - 1/2*x^3*e^a*gamma(3/n,
 -b*x^n)/((-b*x^n)^(3/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \sinh\left(bx^n + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*x**n),x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(bx^n + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(b*x^n + a), x)
```

3.59 $\int x \sinh(a + bx^n)\, dx$

Optimal. Leaf size=75

$$\frac{e^{-a}x^2\left(bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},bx^n\right)}{2n}-\frac{e^{a}x^2\left(-bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-bx^n\right)}{2n}$$

```
[Out] -(E^a*x^2*Gamma[2/n, -(b*x^n)])/(2*n*(-(b*x^n))^(2/n)) + (x^2*Gamma[2/n, b*
x^n])/(2*E^a*n*(b*x^n)^(2/n))
```

Rubi [A] time = 0.0406622, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5360, 2218}

$$\frac{e^{-a}x^2\left(bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},bx^n\right)}{2n}-\frac{e^{a}x^2\left(-bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*x^2*Gamma[2/n, -(b*x^n)])/(2*n*(-(b*x^n))^(2/n)) + (x^2*Gamma[2/n, b*
x^n])/(2*E^a*n*(b*x^n)^(2/n))
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x\sinh(a+bx^n)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}x\,dx\right)+\frac{1}{2}\int e^{a+bx^n}x\,dx$$
$$= -\frac{e^a x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-bx^n\right)}{2n}+\frac{e^{-a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},bx^n\right)}{2n}$$

Mathematica [A] time = 0.0811381, size = 88, normalized size = 1.17

$$-\frac{x^2\left(-b^2x^{2n}\right)^{-2/n}\left((\sinh(a)+\cosh(a))(bx^n)^{2/n}\,\text{Gamma}\left(\frac{2}{n},-bx^n\right)-(\cosh(a)-\sinh(a))(-bx^n)^{2/n}\,\text{Gamma}\left(\frac{2}{n},bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^n],x]
```

```
[Out] -(x^2*(-((-(b*x^n))^(2/n)*Gamma[2/n, b*x^n]*(Cosh[a] - Sinh[a])) + (b*x^n)^
(2/n)*Gamma[2/n, -(b*x^n)]*(Cosh[a] + Sinh[a])))/(2*n*(-(b^2*x^(2*n)))^(2/n
))
```

Maple [C] time = 0.065, size = 69, normalized size = 0.9

$$\frac{x^2\sinh(a)}{2}{}_1F_2(n^{-1};\ \frac{1}{2},1+n^{-1};\ \frac{x^{2\,n}b^2}{4})+\frac{x^{n+2}b\cosh(a)}{n+2}{}_1F_2(\frac{1}{2}+n^{-1};\ \frac{3}{2},\frac{3}{2}+n^{-1};\ \frac{x^{2\,n}b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*x^n),x)
```

```
[Out] 1/2*x^2*hypergeom([1/n],[1/2,1+1/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+2)*x^(n+2
)*b*hypergeom([1/2+1/n],[3/2,3/2+1/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Maxima [A] time = 1.21475, size = 99, normalized size = 1.32

$$\frac{x^2e^{(-a)}\Gamma\left(\frac{2}{n},bx^n\right)}{2\,(bx^n)^{\frac{2}{n}}\,n}-\frac{x^2e^a\Gamma\left(\frac{2}{n},-bx^n\right)}{2\,(-bx^n)^{\frac{2}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*e^(-a)*gamma(2/n, b*x^n)/((b*x^n)^(2/n)*n) - 1/2*x^2*e^a*gamma(2/n,
 -b*x^n)/((-b*x^n)^(2/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x \sinh\left(bx^{n} + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh\left(a + bx^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x**n),x)
```

```
[Out] Integral(x*sinh(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh\left(bx^{n} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x^n + a), x)
```

3.60 $\int \sinh(a + bx^n)\, dx$

Optimal. Leaf size=67

$$\frac{e^{-a}x(bx^n)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n}, bx^n\right)}{2n} - \frac{e^{a}x(-bx^n)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n}, -bx^n\right)}{2n}$$

```
[Out] -(E^a*x*Gamma[n^(-1), -(b*x^n)])/(2*n*(-(b*x^n))^n^(-1)) + (x*Gamma[n^(-1),
 b*x^n])/(2*E^a*n*(b*x^n)^n^(-1))
```

Rubi [A] time = 0.0170829, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5306, 2208}

$$\frac{e^{-a}x(bx^n)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n}, bx^n\right)}{2n} - \frac{e^{a}x(-bx^n)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n}, -bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*x*Gamma[n^(-1), -(b*x^n)])/(2*n*(-(b*x^n))^n^(-1)) + (x*Gamma[n^(-1),
 b*x^n])/(2*E^a*n*(b*x^n)^n^(-1))
```

Rule 5306

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\int \sinh(a+bx^n)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}\,dx\right)+\frac{1}{2}\int e^{a+bx^n}\,dx$$
$$= -\frac{e^a x\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}+\frac{e^{-a}x\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}$$

Mathematica [A] time = 0.0804836, size = 79, normalized size = 1.18

$$\frac{\left(-b^2x^{2n}\right)^{-1/n}\left(x(\cosh(a)-\sinh(a))\left(-bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},bx^n\right)-x(\sinh(a)+\cosh(a))\left(bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},-bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n],x]
```

```
[Out] (x*(-(b*x^n))^n^(-1)*Gamma[n^(-1), b*x^n]*(Cosh[a] - Sinh[a]) - x*(b*x^n)^n
^(-1)*Gamma[n^(-1), -(b*x^n)]*(Cosh[a] + Sinh[a]))/(2*n*(-(b^2*x^(2*n)))^n^
(-1))
```

Maple [C] time = 0.052, size = 74, normalized size = 1.1

$$x_1F_2\left(\frac{1}{2n};\frac{1}{2},1+\frac{1}{2n};\frac{x^{2n}b^2}{4}\right)\sinh(a)+\frac{x^{n+1}b\cosh(a)}{n+1}{}_1F_2\left(\frac{1}{2}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{1}{2n};\frac{x^{2n}b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n),x)
```

```
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+1)*x^(n+1)*
b*hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Maxima [A] time = 1.17206, size = 82, normalized size = 1.22

$$\frac{xe^{(-a)}\Gamma\left(\frac{1}{n},bx^n\right)}{2\left(bx^n\right)^{\left(\frac{1}{n}\right)}n}-\frac{xe^a\Gamma\left(\frac{1}{n},-bx^n\right)}{2\left(-bx^n\right)^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 1/2*x*e^a*gamma(1/n, -b*
x^n)/((-b*x^n)^(1/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sinh\left(bx^n + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n),x)
```

```
[Out] Integral(sinh(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(bx^n + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a), x)
```

3.61 $\int \frac{\sinh(a+bx^n)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\sinh(a)\mathrm{Chi}\,(bx^n)}{n}+\frac{\cosh(a)\mathrm{Shi}\,(bx^n)}{n}$$

```
[Out] (CoshIntegral[b*x^n]*Sinh[a])/n + (Cosh[a]*SinhIntegral[b*x^n])/n
```

Rubi [A] time = 0.0372303, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5318, 5317, 5316}

$$\frac{\sinh(a)\mathrm{Chi}\,(bx^n)}{n}+\frac{\cosh(a)\mathrm{Shi}\,(bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]/x,x]
```

```
[Out] (CoshIntegral[b*x^n]*Sinh[a])/n + (Cosh[a]*SinhIntegral[b*x^n])/n
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sinh(a+bx^n)}{x}\,dx = \cosh(a)\int \frac{\sinh(bx^n)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx^n)}{x}\,dx$$
$$= \frac{\mathbf{Chi}(bx^n)\sinh(a)}{n} + \frac{\cosh(a)\mathbf{Shi}(bx^n)}{n}$$

Mathematica [A] time = 0.0210538, size = 23, normalized size = 0.92

$$\frac{\sinh(a)\mathbf{Chi}(bx^n) + \cosh(a)\mathbf{Shi}(bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]/x,x]
```

```
[Out] (CoshIntegral[b*x^n]*Sinh[a] + Cosh[a]*SinhIntegral[b*x^n])/n
```

Maple [A] time = 0.014, size = 33, normalized size = 1.3

$$\frac{e^{-a}\mathrm{Ei}(1, bx^n)}{2n} - \frac{e^{a}\mathrm{Ei}(1, -bx^n)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)/x,x)
```

```
[Out] 1/2/n*exp(-a)*Ei(1,b*x^n)-1/2/n*exp(a)*Ei(1,-b*x^n)
```

Maxima [A] time = 1.23512, size = 41, normalized size = 1.64

$$-\frac{\mathrm{Ei}(-bx^n)\,e^{(-a)}}{2n} + \frac{\mathrm{Ei}(bx^n)\,e^{a}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*Ei(-b*x^n)*e^(-a)/n + 1/2*Ei(b*x^n)*e^a/n
```

Fricas [B] time = 1.87791, size = 178, normalized size = 7.12

$$\frac{(\cosh(a) + \sinh(a))\mathrm{Ei}(b\cosh(n\log(x)) + b\sinh(n\log(x))) - (\cosh(a) - \sinh(a))\mathrm{Ei}(-b\cosh(n\log(x)) - b\sinh(n\log(x)))}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) - (cosh(a)
 - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a + bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)/x,x)
```

```
[Out] Integral(sinh(a + b*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx^n + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)/x, x)
```

3.62 $\int \frac{\sinh(a+bx^n)}{x^2}\,dx$

Optimal. Leaf size=71

$$\frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\mathrm{Gamma}\left(-\frac{1}{n},bx^n\right)}{2nx}-\frac{e^{a}(-bx^n)^{\frac{1}{n}}\,\mathrm{Gamma}\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

```
[Out] -(E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(2*n*x) + ((b*x^n)^n^(-1)
*Gamma[-n^(-1), b*x^n])/(2*E^a*n*x)
```

Rubi [A] time = 0.0634738, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5360, 2218}

$$\frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\mathrm{Gamma}\left(-\frac{1}{n},bx^n\right)}{2nx}-\frac{e^{a}(-bx^n)^{\frac{1}{n}}\,\mathrm{Gamma}\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]/x^2,x]
```

```
[Out] -(E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(2*n*x) + ((b*x^n)^n^(-1)
*Gamma[-n^(-1), b*x^n])/(2*E^a*n*x)
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx^n)}{x^2}\,dx = -\left(\frac{1}{2}\int \frac{e^{-a-bx^n}}{x^2}\,dx\right) + \frac{1}{2}\int \frac{e^{a+bx^n}}{x^2}\,dx$$
$$= -\frac{e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx} + \frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx}$$

Mathematica [A] time = 0.0672384, size = 68, normalized size = 0.96

$$\frac{(\cosh(a)-\sinh(a))\,(bx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},bx^n\right)-(\sinh(a)+\cosh(a))\,(-bx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]/x^2,x]
```

```
[Out] ((b*x^n)^n^(-1)*Gamma[-n^(-1), b*x^n]*(Cosh[a] - Sinh[a]) - (-(b*x^n))^n^(-
1)*Gamma[-n^(-1), -(b*x^n)]*(Cosh[a] + Sinh[a]))/(2*n*x)
```

Maple [C] time = 0.077, size = 77, normalized size = 1.1

$$-\frac{\sinh(a)}{x}\,{}_1F_2\left(-\frac{1}{2n};\,\frac{1}{2},1-\frac{1}{2n};\,\frac{x^{2n}b^2}{4}\right)+\frac{x^{-1+n}b\cosh(a)}{-1+n}\,{}_1F_2\left(\frac{1}{2}-\frac{1}{2n};\,\frac{3}{2},\frac{3}{2}-\frac{1}{2n};\,\frac{x^{2n}b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)/x^2,x)
```

```
[Out] -1/x*hypergeom([-1/2/n],[1/2,1-1/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(-1+n)*x^(
-1+n)*b*hypergeom([1/2-1/2/n],[3/2,3/2-1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Maxima [A] time = 1.19774, size = 88, normalized size = 1.24

$$\frac{(bx^n)^{\left(\frac{1}{n}\right)}e^{(-a)}\Gamma\left(-\frac{1}{n},bx^n\right)}{2\,nx}-\frac{(-bx^n)^{\left(\frac{1}{n}\right)}e^{a}\Gamma\left(-\frac{1}{n},-bx^n\right)}{2\,nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^n)^(1/n)*e^(-a)*gamma(-1/n, b*x^n)/(n*x) - 1/2*(-b*x^n)^(1/n)*e^a*
gamma(-1/n, -b*x^n)/(n*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(bx^n+a\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+bx^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^n+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)/x^2, x)
```

3.63 $\int \frac{\sinh(a+bx^n)}{x^3} dx$

Optimal. Leaf size=75

$$\frac{e^{-a}(bx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},bx^n\right)}{2nx^2}-\frac{e^{a}(-bx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

```
[Out] -(E^a*(-(b*x^n))^(2/n)*Gamma[-2/n, -(b*x^n)])/(2*n*x^2) + ((b*x^n)^(2/n)*Ga
mma[-2/n, b*x^n])/(2*E^a*n*x^2)
```

Rubi [A] time = 0.062377, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5360, 2218}

$$\frac{e^{-a}(bx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},bx^n\right)}{2nx^2}-\frac{e^{a}(-bx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]/x^3,x]
```

```
[Out] -(E^a*(-(b*x^n))^(2/n)*Gamma[-2/n, -(b*x^n)])/(2*n*x^2) + ((b*x^n)^(2/n)*Ga
mma[-2/n, b*x^n])/(2*E^a*n*x^2)
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = -\left(\frac{1}{2}\int \frac{e^{-a-bx^n}}{x^3}\,dx\right) + \frac{1}{2}\int \frac{e^{a+bx^n}}{x^3}\,dx$$
$$= -\frac{e^a(-bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2} + \frac{e^{-a}(bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2}$$

Mathematica [A] time = 0.0717494, size = 72, normalized size = 0.96

$$\frac{(\cosh(a)-\sinh(a))(bx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},bx^n\right)-(\sinh(a)+\cosh(a))(-bx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]/x^3,x]
```

```
[Out] ((b*x^n)^(2/n)*Gamma[-2/n, b*x^n]*(Cosh[a] - Sinh[a]) - (-(b*x^n))^(2/n)*Ga
mma[-2/n, -(b*x^n)]*(Cosh[a] + Sinh[a]))/(2*n*x^2)
```

Maple [C] time = 0.039, size = 77, normalized size = 1.

$$-\frac{\sinh(a)}{2x^2}\,{}_1F_2(-n^{-1};\ \frac{1}{2},1-n^{-1};\ \frac{x^{2n}b^2}{4}) + \frac{x^{-2+n}b\cosh(a)}{-2+n}\,{}_1F_2(\frac{1}{2}-n^{-1};\ \frac{3}{2},\frac{3}{2}-n^{-1};\ \frac{x^{2n}b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)/x^3,x)
```

```
[Out] -1/2/x^2*hypergeom([-1/n],[1/2,1-1/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(-2+n)*x^(
-2+n)*b*hypergeom([1/2-1/n],[3/2,3/2-1/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Maxima [A] time = 1.1991, size = 93, normalized size = 1.24

$$\frac{(bx^n)^{\frac{2}{n}}\,e^{(-a)}\Gamma\left(-\frac{2}{n},bx^n\right)}{2\,nx^2} - \frac{(-bx^n)^{\frac{2}{n}}\,e^{a}\Gamma\left(-\frac{2}{n},-bx^n\right)}{2\,nx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^n)^(2/n)*e^(-a)*gamma(-2/n, b*x^n)/(n*x^2) - 1/2*(-b*x^n)^(2/n)*e^
a*gamma(-2/n, -b*x^n)/(n*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(bx^n+a\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+bx^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**n)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(bx^n+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)/x^3, x)
```

3.64 $\int x^2 \sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=99

$$-\frac{e^{2a}2^{-\frac{3}{n}-2}x^3(-bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{3}{n}-2}x^3(bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},2bx^n\right)}{n}-\frac{x^3}{6}$$

```
[Out] -x^3/6 - (2^(-2 - 3/n)*E^(2*a)*x^3*Gamma[3/n, -2*b*x^n])/(n*(-(b*x^n))^(3/n
)) - (2^(-2 - 3/n)*x^3*Gamma[3/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(3/n))
```

Rubi [A] time = 0.136861, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}2^{-\frac{3}{n}-2}x^3(-bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{3}{n}-2}x^3(bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},2bx^n\right)}{n}-\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^n]^2,x]
```

```
[Out] -x^3/6 - (2^(-2 - 3/n)*E^(2*a)*x^3*Gamma[3/n, -2*b*x^n])/(n*(-(b*x^n))^(3/n
)) - (2^(-2 - 3/n)*x^3*Gamma[3/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(3/n))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh^2(a+bx^n)\, dx &= \int \left(-\frac{x^2}{2}+\frac{1}{2}x^2\cosh(2a+2bx^n)\right) dx\\
&= -\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh(2a+2bx^n)\, dx\\
&= -\frac{x^3}{6}+\frac{1}{4}\int e^{-2a-2bx^n}x^2\, dx+\frac{1}{4}\int e^{2a+2bx^n}x^2\, dx\\
&= -\frac{x^3}{6}-\frac{2^{-2-\frac{3}{n}}e^{2a}x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{3}{n}}e^{-2a}x^3(bx^n)^{-3/n}\Gamma\left(\frac{3}{n},2bx^n\right)}{n}
\end{aligned}$$

Mathematica [A] time = 1.43503, size = 89, normalized size = 0.9

$$-\frac{x^3\left(3e^{2a}8^{-1/n}(-bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-2bx^n\right)+3e^{-2a}8^{-1/n}(bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},2bx^n\right)+2n\right)}{12n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^n]^2,x]
```

```
[Out] -(x^3*(2*n + (3*E^(2*a)*Gamma[3/n, -2*b*x^n])/(8^n^(-1)*(-(b*x^n))^(3/n)) +
 (3*Gamma[3/n, 2*b*x^n])/(8^n^(-1)*E^(2*a)*(b*x^n)^(3/n))))/(12*n)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^2(\sinh(a+bx^n))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*x^n)^2,x)
```

```
[Out] int(x^2*sinh(a+b*x^n)^2,x)
```

Maxima [A] time = 1.22367, size = 111, normalized size = 1.12

$$-\frac{1}{6}x^3 - \frac{x^3e^{(-2a)}\Gamma\left(\frac{3}{n}, 2bx^n\right)}{4(2bx^n)^{\frac{3}{n}}n} - \frac{x^3e^{(2a)}\Gamma\left(\frac{3}{n}, -2bx^n\right)}{4(-2bx^n)^{\frac{3}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^3 - 1/4*x^3*e^(-2*a)*gamma(3/n, 2*b*x^n)/((2*b*x^n)^(3/n)*n) - 1/4*x
^3*e^(2*a)*gamma(3/n, -2*b*x^n)/((-2*b*x^n)^(3/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\sinh(bx^n + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x^n + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sinh^2(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sinh(bx^n + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(b*x^n + a)^2, x)
```

3.65 $\int x \sinh^2(a + bx^n)\,dx$

Optimal. Leaf size=99

$$-\frac{e^{2a}4^{-\frac{1}{n}-1}x^2\left(-bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{e^{-2a}4^{-\frac{1}{n}-1}x^2\left(bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},2bx^n\right)}{n}-\frac{x^2}{4}$$

```
[Out] -x^2/4 - (4^(-1 - n^(-1))*E^(2*a)*x^2*Gamma[2/n, -2*b*x^n])/(n*(-(b*x^n))^(
2/n)) - (4^(-1 - n^(-1))*x^2*Gamma[2/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(2/n))
```

Rubi [A] time = 0.107395, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}4^{-\frac{1}{n}-1}x^2\left(-bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{e^{-2a}4^{-\frac{1}{n}-1}x^2\left(bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},2bx^n\right)}{n}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^n]^2,x]
```

```
[Out] -x^2/4 - (4^(-1 - n^(-1))*E^(2*a)*x^2*Gamma[2/n, -2*b*x^n])/(n*(-(b*x^n))^(
2/n)) - (4^(-1 - n^(-1))*x^2*Gamma[2/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(2/n))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh^2\left(a+bx^n\right)\,dx &= \int \left(-\frac{x}{2}+\frac{1}{2}x\cosh\left(2a+2bx^n\right)\right)dx\\
&= -\frac{x^2}{4}+\frac{1}{2}\int x\cosh\left(2a+2bx^n\right)\,dx\\
&= -\frac{x^2}{4}+\frac{1}{4}\int e^{-2a-2bx^n}x\,dx+\frac{1}{4}\int e^{2a+2bx^n}x\,dx\\
&= -\frac{x^2}{4}-\frac{4^{-1-\frac{1}{n}}e^{2a}x^2\left(-bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{4^{-1-\frac{1}{n}}e^{-2a}x^2\left(bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},2bx^n\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 1.24474, size = 85, normalized size = 0.86

$$-\frac{x^2\left(e^{2a}4^{-1/n}\left(-bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-2bx^n\right)+e^{-2a}4^{-1/n}\left(bx^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},2bx^n\right)+n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^n]^2,x]
```

```
[Out] -(x^2*(n + (E^(2*a)*Gamma[2/n, -2*b*x^n])/(4^n^(-1)*(-(b*x^n))^(2/n)) + Gam
ma[2/n, 2*b*x^n]/(4^n^(-1)*E^(2*a)*(b*x^n)^(2/n))))/(4*n)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x\left(\sinh\left(a+bx^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*x^n)^2,x)
```

```
[Out] int(x*sinh(a+b*x^n)^2,x)
```

Maxima [A] time = 1.18564, size = 111, normalized size = 1.12

$$-\frac{1}{4}x^2 - \frac{x^2e^{(-2a)}\Gamma\left(\frac{2}{n}, 2bx^n\right)}{4(2bx^n)^{\frac{2}{n}}n} - \frac{x^2e^{(2a)}\Gamma\left(\frac{2}{n}, -2bx^n\right)}{4(-2bx^n)^{\frac{2}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^2 - 1/4*x^2*e^(-2*a)*gamma(2/n, 2*b*x^n)/((2*b*x^n)^(2/n)*n) - 1/4*x
^2*e^(2*a)*gamma(2/n, -2*b*x^n)/((-2*b*x^n)^(2/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\sinh(bx^n + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x^n + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh^2(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh(bx^n + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x^n + a)^2, x)
```

3.66 $\int \sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=89

$$-\frac{e^{2a}2^{-\frac{1}{n}-2}x\left(-bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{1}{n}-2}x\left(bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2bx^n\right)}{n}-\frac{x}{2}$$

```
[Out] -x/2 - (2^(-2 - n^(-1))*E^(2*a)*x*Gamma[n^(-1), -2*b*x^n])/(n*(-(b*x^n))^n^
(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), 2*b*x^n])/(E^(2*a)*n*(b*x^n)^n^(-1
))
```

Rubi [A] time = 0.0649043, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5308, 5307, 2208}

$$-\frac{e^{2a}2^{-\frac{1}{n}-2}x\left(-bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{1}{n}-2}x\left(bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2bx^n\right)}{n}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^2,x]
```

```
[Out] -x/2 - (2^(-2 - n^(-1))*E^(2*a)*x*Gamma[n^(-1), -2*b*x^n])/(n*(-(b*x^n))^n^
(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), 2*b*x^n])/(E^(2*a)*n*(b*x^n)^n^(-1
))
```

Rule 5308

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rule 5307

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
```

```
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \sinh^2\left(a+bx^n\right)\,dx &= \int \left(-\frac{1}{2}+\frac{1}{2}\cosh\left(2a+2bx^n\right)\right)dx\\ &= -\frac{x}{2}+\frac{1}{2}\int \cosh\left(2a+2bx^n\right)\,dx\\ &= -\frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^n}\,dx+\frac{1}{4}\int e^{2a+2bx^n}\,dx\\ &= -\frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2a}x\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2a}x\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2bx^n\right)}{n}\end{aligned}$$

Mathematica [A] time = 1.07465, size = 81, normalized size = 0.91

$$-\frac{x\left(e^{2a}2^{-1/n}\left(-bx^n\right)^{-1/n}\,\mathrm{Gamma}\left(\frac{1}{n},-2bx^n\right)+e^{-2a}2^{-1/n}\left(bx^n\right)^{-1/n}\,\mathrm{Gamma}\left(\frac{1}{n},2bx^n\right)+2n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^2,x]
```

```
[Out] -(x*(2*n + (E^(2*a)*Gamma[n^(-1), -2*b*x^n])/(2^n^(-1)*(-(b*x^n))^n^(-1)) +
 Gamma[n^(-1), 2*b*x^n]/(2^n^(-1)*E^(2*a)*(b*x^n)^n^(-1))))/(4*n)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(\sinh\left(a+bx^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^2,x)
```

```
[Out] int(sinh(a+b*x^n)^2,x)
```

Maxima [A] time = 1.16296, size = 92, normalized size = 1.03

$$-\frac{1}{2}\,x-\frac{xe^{(-2\,a)}\Gamma\left(\frac{1}{n},2\,bx^{n}\right)}{4\,\left(2\,bx^{n}\right)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{(2\,a)}\Gamma\left(\frac{1}{n},-2\,bx^{n}\right)}{4\,\left(-2\,bx^{n}\right)^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/4*x*e^(-2*a)*gamma(1/n, 2*b*x^n)/((2*b*x^n)^(1/n)*n) - 1/4*x*e^(
2*a)*gamma(1/n, -2*b*x^n)/((-2*b*x^n)^(1/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sinh\left(bx^{n}+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{2}\left(a+bx^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(bx^{n}+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^2, x)
```

3.67 $\int \frac{\sinh^2(a+bx^n)}{x} dx$

Optimal. Leaf size=43

$$\frac{\cosh(2a)\text{Chi}\left(2bx^n\right)}{2n}+\frac{\sinh(2a)\text{Shi}\left(2bx^n\right)}{2n}-\frac{\log(x)}{2}$$

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n])/(2*n) - Log[x]/2 + (Sinh[2*a]*SinhIntegra
l[2*b*x^n])/(2*n)
```

Rubi [A] time = 0.0639129, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5362, 5319, 5317, 5316}

$$\frac{\cosh(2a)\text{Chi}\left(2bx^n\right)}{2n}+\frac{\sinh(2a)\text{Shi}\left(2bx^n\right)}{2n}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n])/(2*n) - Log[x]/2 + (Sinh[2*a]*SinhIntegra
l[2*b*x^n])/(2*n)
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5319

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx^n)}{x}\,dx &= \int \left(-\frac{1}{2x}+\frac{\cosh(2a+2bx^n)}{2x}\right)dx \\
&= -\frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh(2a+2bx^n)}{x}\,dx \\
&= -\frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh(2bx^n)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh(2bx^n)}{x}\,dx \\
&= \frac{\cosh(2a)\mathbf{Chi}(2bx^n)}{2n}-\frac{\log(x)}{2}+\frac{\sinh(2a)\mathbf{Shi}(2bx^n)}{2n}
\end{aligned}$$

Mathematica [A] time = 0.0304173, size = 39, normalized size = 0.91

$$\frac{\cosh(2a)\mathbf{Chi}(2bx^n)+\sinh(2a)\mathbf{Shi}(2bx^n)}{2n}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^2/x,x]
```

```
[Out] -Log[x]/2 + (Cosh[2*a]*CoshIntegral[2*b*x^n] + Sinh[2*a]*SinhIntegral[2*b*x
^n])/(2*n)
```

Maple [A] time = 0.079, size = 40, normalized size = 0.9

$$-\frac{\ln(x)}{2}-\frac{e^{-2a}\mathrm{Ei}(1,2bx^n)}{4n}-\frac{e^{2a}\mathrm{Ei}(1,-2bx^n)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^2/x,x)
```

```
[Out] -1/2*ln(x)-1/4/n*exp(-2*a)*Ei(1,2*b*x^n)-1/4/n*exp(2*a)*Ei(1,-2*b*x^n)
```

Maxima [A] time = 1.1796, size = 50, normalized size = 1.16

$$\frac{\operatorname{Ei}\left(2\,bx^{n}\right)e^{(2\,a)}}{4\,n}+\frac{\operatorname{Ei}\left(-2\,bx^{n}\right)e^{(-2\,a)}}{4\,n}-\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*b*x^n)*e^(2*a)/n + 1/4*Ei(-2*b*x^n)*e^(-2*a)/n - 1/2*log(x)
```

Fricas [A] time = 1.8706, size = 217, normalized size = 5.05

$$\frac{(\cosh\left(2\,a\right)+\sinh\left(2\,a\right))\operatorname{Ei}\left(2\,b\cosh\left(n\log\left(x\right)\right)+2\,b\sinh\left(n\log\left(x\right)\right)\right)+(\cosh\left(2\,a\right)-\sinh\left(2\,a\right))\operatorname{Ei}\left(-2\,b\cosh\left(n\log\left(x\right)\right)-\right.}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*((cosh(2*a) + sinh(2*a))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) +
(cosh(2*a) - sinh(2*a))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - 2*n*
log(x))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{2}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x**n)**2/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh{(bx^n + a)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^2/x, x)
```

3.68 $\int \frac{\sinh^2(a+bx^n)}{x^2} dx$

Optimal. Leaf size=91

$$-\frac{e^{2a}2^{\frac{1}{n}-2}(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{e^{-2a}2^{\frac{1}{n}-2}(bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},2bx^n\right)}{nx}+\frac{1}{2x}$$

```
[Out] 1/(2*x) - (2^(-2 + n^(-1))*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^
n])/(n*x) - (2^(-2 + n^(-1))*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/(E^(2*
a)*n*x)
```

Rubi [A] time = 0.128284, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}2^{\frac{1}{n}-2}(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{e^{-2a}2^{\frac{1}{n}-2}(bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},2bx^n\right)}{nx}+\frac{1}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^2/x^2,x]
```

```
[Out] 1/(2*x) - (2^(-2 + n^(-1))*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^
n])/(n*x) - (2^(-2 + n^(-1))*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/(E^(2*
a)*n*x)
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx &= \int \left(-\frac{1}{2x^2}+\frac{\cosh(2a+2bx^n)}{2x^2}\right)dx\\
&= \frac{1}{2x}+\frac{1}{2}\int \frac{\cosh(2a+2bx^n)}{x^2}\,dx\\
&= \frac{1}{2x}+\frac{1}{4}\int \frac{e^{-2a-2bx^n}}{x^2}\,dx+\frac{1}{4}\int \frac{e^{2a+2bx^n}}{x^2}\,dx\\
&= \frac{1}{2x}-\frac{2^{-2+\frac{1}{n}}e^{2a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{2^{-2+\frac{1}{n}}e^{-2a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2bx^n\right)}{nx}
\end{aligned}$$

Mathematica [A] time = 1.37531, size = 79, normalized size = 0.87

$$-\frac{e^{2a}2^{\frac{1}{n}}(-bx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},-2bx^n\right)+e^{-2a}2^{\frac{1}{n}}(bx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},2bx^n\right)-2n}{4nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^2/x^2,x]
```

```
[Out] -(-2*n + 2^n^(-1)*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^n] + (2^n
^(-1)*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/E^(2*a))/(4*n*x)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(\sinh(a+bx^n))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^2/x^2,x)
```

```
[Out] int(sinh(a+b*x^n)^2/x^2,x)
```

Maxima [A] time = 1.21826, size = 100, normalized size = 1.1

$$-\frac{(2\,bx^n)^{\left(\frac{1}{n}\right)}e^{(-2\,a)}\Gamma\left(-\frac{1}{n},2\,bx^n\right)}{4\,nx}-\frac{(-2\,bx^n)^{\left(\frac{1}{n}\right)}e^{(2\,a)}\Gamma\left(-\frac{1}{n},-2\,bx^n\right)}{4\,nx}+\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^n)^(1/n)*e^(-2*a)*gamma(-1/n, 2*b*x^n)/(n*x) - 1/4*(-2*b*x^n)^(
1/n)*e^(2*a)*gamma(-1/n, -2*b*x^n)/(n*x) + 1/2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(bx^n+a\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^2\left(a+bx^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**2/x**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^{n}+a\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^2/x^2, x)
```

3.69 $\int x^2 \sinh^3 (a + bx^n) \, dx$

Optimal. Leaf size=166

$$-\frac{e^{3a}3^{-3/n}x^3 (-bx^n)^{-3/n} \text{Gamma}\left(\frac{3}{n}, -3bx^n\right)}{8n} + \frac{3e^a x^3 (-bx^n)^{-3/n} \text{Gamma}\left(\frac{3}{n}, -bx^n\right)}{8n} - \frac{3e^{-a}x^3 (bx^n)^{-3/n} \text{Gamma}\left(\frac{3}{n}, bx^n\right)}{8n}$$

```
[Out] -(E^(3*a)*x^3*Gamma[3/n, -3*b*x^n])/(8*3^(3/n)*n*(-(b*x^n))^(3/n)) + (3*E^a
*x^3*Gamma[3/n, -(b*x^n)])/(8*n*(-(b*x^n))^(3/n)) - (3*x^3*Gamma[3/n, b*x^n
])/(8*E^a*n*(b*x^n)^(3/n)) + (x^3*Gamma[3/n, 3*b*x^n])/(8*3^(3/n)*E^(3*a)*n
*(b*x^n)^(3/n))
```

Rubi [A] time = 0.195389, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}3^{-3/n}x^3 (-bx^n)^{-3/n} \text{Gamma}\left(\frac{3}{n}, -3bx^n\right)}{8n} + \frac{3e^a x^3 (-bx^n)^{-3/n} \text{Gamma}\left(\frac{3}{n}, -bx^n\right)}{8n} - \frac{3e^{-a}x^3 (bx^n)^{-3/n} \text{Gamma}\left(\frac{3}{n}, bx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*x^3*Gamma[3/n, -3*b*x^n])/(8*3^(3/n)*n*(-(b*x^n))^(3/n)) + (3*E^a
*x^3*Gamma[3/n, -(b*x^n)])/(8*n*(-(b*x^n))^(3/n)) - (3*x^3*Gamma[3/n, b*x^n
])/(8*E^a*n*(b*x^n)^(3/n)) + (x^3*Gamma[3/n, 3*b*x^n])/(8*3^(3/n)*E^(3*a)*n
*(b*x^n)^(3/n))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sinh^3(a+bx^n)\,dx &= \int\left(-\frac{3}{4}x^2\sinh(a+bx^n)+\frac{1}{4}x^2\sinh(3a+3bx^n)\right)dx\\ &= \frac{1}{4}\int x^2\sinh(3a+3bx^n)\,dx-\frac{3}{4}\int x^2\sinh(a+bx^n)\,dx\\ &= -\left(\frac{1}{8}\int e^{-3a-3bx^n}x^2\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}x^2\,dx+\frac{3}{8}\int e^{-a-bx^n}x^2\,dx-\frac{3}{8}\int e^{a+bx^n}x^2\,dx\\ &= -\frac{3^{-3/n}e^{3a}x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-3bx^n\right)}{8n}+\frac{3e^ax^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^3(bx^n)^{-3/n}\Gamma\left(\frac{3}{n},\right.}{8n}\end{aligned}$$

Mathematica [A] time = 1.5339, size = 161, normalized size = 0.97

$$-\frac{e^{-3a}27^{-1/n}x^3\left(-b^2x^{2n}\right)^{-3/n}\left((-bx^n)^{3/n}\left(e^{2a}3^{\frac{n+3}{n}}\text{Gamma}\left(\frac{3}{n},bx^n\right)-\text{Gamma}\left(\frac{3}{n},3bx^n\right)\right)+e^{6a}(bx^n)^{3/n}\text{Gamma}\left(\frac{3}{n},-3bx\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(x^3*(E^(6*a)*(b*x^n)^(3/n)*Gamma[3/n, -3*b*x^n] - 3^((3 + n)/n)*E^(4*a)*(
b*x^n)^(3/n)*Gamma[3/n, -(b*x^n)] + (-(b*x^n))^(3/n)*(3^((3 + n)/n)*E^(2*a)
*Gamma[3/n, b*x^n] - Gamma[3/n, 3*b*x^n])))/(8*27^n^(-1)*E^(3*a)*n*(-(b^2*x
^(2*n)))^(3/n))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int x^2\left(\sinh(a+bx^n)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*x^n)^3,x)
```

```
[Out] int(x^2*sinh(a+b*x^n)^3,x)
```

Maxima [A] time = 1.34138, size = 201, normalized size = 1.21

$$\frac{x^3e^{(-3a)}\Gamma\left(\frac{3}{n}, 3bx^n\right)}{8\left(3bx^n\right)^{\frac{3}{n}}n} - \frac{3x^3e^{(-a)}\Gamma\left(\frac{3}{n}, bx^n\right)}{8\left(bx^n\right)^{\frac{3}{n}}n} + \frac{3x^3e^{a}\Gamma\left(\frac{3}{n}, -bx^n\right)}{8\left(-bx^n\right)^{\frac{3}{n}}n} - \frac{x^3e^{(3a)}\Gamma\left(\frac{3}{n}, -3bx^n\right)}{8\left(-3bx^n\right)^{\frac{3}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x^3*e^(-3*a)*gamma(3/n, 3*b*x^n)/((3*b*x^n)^(3/n)*n) - 3/8*x^3*e^(-a)*g
amma(3/n, b*x^n)/((b*x^n)^(3/n)*n) + 3/8*x^3*e^a*gamma(3/n, -b*x^n)/((-b*x^
n)^(3/n)*n) - 1/8*x^3*e^(3*a)*gamma(3/n, -3*b*x^n)/((-3*b*x^n)^(3/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\sinh\left(bx^n + a\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x^n + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sinh^3\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*x**n)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh{(bx^n + a)}^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(b*x^n + a)^3, x)
```

3.70 $\int x \sinh^3(a+bx^n)\,dx$

Optimal. Leaf size=166

$$-\frac{e^{3a}9^{-1/n}x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^a x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^2(bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},bx^n\right)}{8n}$$

```
[Out] -(E^(3*a)*x^2*Gamma[2/n, -3*b*x^n])/(8*9^n^(-1)*n*(-(b*x^n))^(2/n)) + (3*E^
a*x^2*Gamma[2/n, -(b*x^n)])/(8*n*(-(b*x^n))^(2/n)) - (3*x^2*Gamma[2/n, b*x^
n])/(8*E^a*n*(b*x^n)^(2/n)) + (x^2*Gamma[2/n, 3*b*x^n])/(8*9^n^(-1)*E^(3*a)
*n*(b*x^n)^(2/n))
```

Rubi [A] time = 0.13314, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}9^{-1/n}x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^a x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^2(bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},bx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*x^2*Gamma[2/n, -3*b*x^n])/(8*9^n^(-1)*n*(-(b*x^n))^(2/n)) + (3*E^
a*x^2*Gamma[2/n, -(b*x^n)])/(8*n*(-(b*x^n))^(2/n)) - (3*x^2*Gamma[2/n, b*x^
n])/(8*E^a*n*(b*x^n)^(2/n)) + (x^2*Gamma[2/n, 3*b*x^n])/(8*9^n^(-1)*E^(3*a)
*n*(b*x^n)^(2/n))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^3(a+bx^n)\,dx &= \int\left(-\frac{3}{4}x\sinh(a+bx^n)+\frac{1}{4}x\sinh(3a+3bx^n)\right)dx\\
&= \frac{1}{4}\int x\sinh(3a+3bx^n)\,dx-\frac{3}{4}\int x\sinh(a+bx^n)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^n}x\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}x\,dx+\frac{3}{8}\int e^{-a-bx^n}x\,dx-\frac{3}{8}\int e^{a+bx^n}x\,dx\\
&= -\frac{9^{-1/n}e^{3a}x^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^ax^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^2(bx^n)^{-2/n}\Gamma\left(\frac{2}{n},b\right.}{8n}
\end{aligned}$$

Mathematica [A] time = 1.55575, size = 161, normalized size = 0.97

$$-\frac{e^{-3a}9^{-1/n}x^2\left(-b^2x^{2n}\right)^{-2/n}\left((-bx^n)^{2/n}\left(e^{2a}3^{\frac{n+2}{n}}\text{Gamma}\left(\frac{2}{n},bx^n\right)-\text{Gamma}\left(\frac{2}{n},3bx^n\right)\right)+e^{6a}(bx^n)^{2/n}\text{Gamma}\left(\frac{2}{n},-3bx^n\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(x^2*(E^(6*a)*(b*x^n)^(2/n)*Gamma[2/n, -3*b*x^n] - 3^((2 + n)/n)*E^(4*a)*(
b*x^n)^(2/n)*Gamma[2/n, -(b*x^n)] + (-(b*x^n))^(2/n)*(3^((2 + n)/n)*E^(2*a)
*Gamma[2/n, b*x^n] - Gamma[2/n, 3*b*x^n])))/(8*9^n^(-1)*E^(3*a)*n*(-(b^2*x^
(2*n)))^(2/n))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x(\sinh(a+bx^n))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*x^n)^3,x)
```

```
[Out] int(x*sinh(a+b*x^n)^3,x)
```

Maxima [A] time = 1.27054, size = 201, normalized size = 1.21

$$\frac{x^2e^{(-3a)}\Gamma\left(\frac{2}{n},3bx^n\right)}{8(3bx^n)^{\frac{2}{n}}n}-\frac{3x^2e^{(-a)}\Gamma\left(\frac{2}{n},bx^n\right)}{8(bx^n)^{\frac{2}{n}}n}+\frac{3x^2e^a\Gamma\left(\frac{2}{n},-bx^n\right)}{8(-bx^n)^{\frac{2}{n}}n}-\frac{x^2e^{(3a)}\Gamma\left(\frac{2}{n},-3bx^n\right)}{8(-3bx^n)^{\frac{2}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x^2*e^(-3*a)*gamma(2/n, 3*b*x^n)/((3*b*x^n)^(2/n)*n) - 3/8*x^2*e^(-a)*g
amma(2/n, b*x^n)/((b*x^n)^(2/n)*n) + 3/8*x^2*e^a*gamma(2/n, -b*x^n)/((-b*x^
n)^(2/n)*n) - 1/8*x^2*e^(3*a)*gamma(2/n, -3*b*x^n)/((-3*b*x^n)^(2/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\sinh\left(bx^n+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x^n + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh^3\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x**n)**3,x)
```

```
[Out] Integral(x*sinh(a + b*x**n)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sinh \left(bx^n + a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x^n + a)^3, x)
```

3.71 $\int \sinh^3(a + bx^n)\, dx$

Optimal. Leaf size=150

$$-\frac{e^{3a}3^{-1/n}x\left(-bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^{a}x\left(-bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x\left(bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},bx^n\right)}{8n}+\frac{e}{}$$

```
[Out] -(E^(3*a)*x*Gamma[n^(-1), -3*b*x^n])/(8*3^n^(-1)*n*(-(b*x^n))^n^(-1)) + (3*
E^a*x*Gamma[n^(-1), -(b*x^n)])/(8*n*(-(b*x^n))^n^(-1)) - (3*x*Gamma[n^(-1),
 b*x^n])/(8*E^a*n*(b*x^n)^n^(-1)) + (x*Gamma[n^(-1), 3*b*x^n])/(8*3^n^(-1)*
E^(3*a)*n*(b*x^n)^n^(-1))
```

Rubi [A] time = 0.0755867, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5308, 5306, 2208}

$$-\frac{e^{3a}3^{-1/n}x\left(-bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^{a}x\left(-bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x\left(bx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},bx^n\right)}{8n}+\frac{e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*x*Gamma[n^(-1), -3*b*x^n])/(8*3^n^(-1)*n*(-(b*x^n))^n^(-1)) + (3*
E^a*x*Gamma[n^(-1), -(b*x^n)])/(8*n*(-(b*x^n))^n^(-1)) - (3*x*Gamma[n^(-1),
 b*x^n])/(8*E^a*n*(b*x^n)^n^(-1)) + (x*Gamma[n^(-1), 3*b*x^n])/(8*3^n^(-1)*
E^(3*a)*n*(b*x^n)^n^(-1))
```

Rule 5308

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rule 5306

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^3(a+bx^n)\,dx &= \int \left(-\frac{3}{4}\sinh(a+bx^n)+\frac{1}{4}\sinh(3a+3bx^n)\right)dx\\
&= \frac{1}{4}\int \sinh(3a+3bx^n)\,dx-\frac{3}{4}\int \sinh(a+bx^n)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^n}\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}\,dx+\frac{3}{8}\int e^{-a-bx^n}\,dx-\frac{3}{8}\int e^{a+bx^n}\,dx\\
&= -\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^{a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{8n}
\end{aligned}
$$

Mathematica [A] time = 1.20334, size = 140, normalized size = 0.93

$$
\frac{e^{-3a}3^{-1/n}x\left(-b^2x^{2n}\right)^{-1/n}\left((-bx^n)^{\frac{1}{n}}\left(\text{Gamma}\left(\frac{1}{n},3bx^n\right)-e^{2a}3^{\frac{1}{n}+1}\text{Gamma}\left(\frac{1}{n},bx^n\right)\right)-e^{6a}(bx^n)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},-3bx^n\right)+e\right.}{8n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^3,x]
```

```
[Out] (x*(-(E^(6*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -3*b*x^n]) + 3^(1 + n^(-1))*E^(4
*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -(b*x^n)] + (-(b*x^n))^n^(-1)*(-(3^(1 + n^
(-1))*E^(2*a)*Gamma[n^(-1), b*x^n]) + Gamma[n^(-1), 3*b*x^n])))/(8*3^n^(-1)
*E^(3*a)*n*(-(b^2*x^(2*n)))^n^(-1))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int (\sinh(a+bx^n))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^3,x)
```

```
[Out] int(sinh(a+b*x^n)^3,x)
```

Maxima [A] time = 1.252, size = 169, normalized size = 1.13

$$\frac{xe^{(-3\,a)}\Gamma\left(\frac{1}{n},3\,bx^n\right)}{8\,(3\,bx^n)^{\left(\frac{1}{n}\right)}n}-\frac{3\,xe^{(-a)}\Gamma\left(\frac{1}{n},bx^n\right)}{8\,(bx^n)^{\left(\frac{1}{n}\right)}n}+\frac{3\,xe^{a}\Gamma\left(\frac{1}{n},-bx^n\right)}{8\,(-bx^n)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{(3\,a)}\Gamma\left(\frac{1}{n},-3\,bx^n\right)}{8\,(-3\,bx^n)^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x*e^(-3*a)*gamma(1/n, 3*b*x^n)/((3*b*x^n)^(1/n)*n) - 3/8*x*e^(-a)*gamma
(1/n, b*x^n)/((b*x^n)^(1/n)*n) + 3/8*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^(1/
n)*n) - 1/8*x*e^(3*a)*gamma(1/n, -3*b*x^n)/((-3*b*x^n)^(1/n)*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sinh\left(bx^n+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^3\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**3,x)
```

```
[Out] Integral(sinh(a + b*x**n)**3, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^3, x)
```

3.72 $\int \frac{\sinh^3(a+bx^n)}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{3\sinh(a)\text{Chi}\left(bx^n\right)}{4n}+\frac{\sinh(3a)\text{Chi}\left(3bx^n\right)}{4n}-\frac{3\cosh(a)\text{Shi}\left(bx^n\right)}{4n}+\frac{\cosh(3a)\text{Shi}\left(3bx^n\right)}{4n}$$

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a])/(4*n) + (CoshIntegral[3*b*x^n]*Sinh[3*a])/
(4*n) - (3*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (Cosh[3*a]*SinhIntegral[3*b
*x^n])/(4*n)
```

Rubi [A] time = 0.0998284, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5362, 5318, 5317, 5316}

$$-\frac{3\sinh(a)\text{Chi}\left(bx^n\right)}{4n}+\frac{\sinh(3a)\text{Chi}\left(3bx^n\right)}{4n}-\frac{3\cosh(a)\text{Shi}\left(bx^n\right)}{4n}+\frac{\cosh(3a)\text{Shi}\left(3bx^n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a])/(4*n) + (CoshIntegral[3*b*x^n]*Sinh[3*a])/
(4*n) - (3*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (Cosh[3*a]*SinhIntegral[3*b
*x^n])/(4*n)
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(a+bx^n)}{x}\,dx &= \int \left(-\frac{3\sinh(a+bx^n)}{4x}+\frac{\sinh(3a+3bx^n)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx^n)}{x}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^n)}{x}\,dx\\ &= -\left(\frac{1}{4}(3\cosh(a))\int \frac{\sinh(bx^n)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx^n)}{x}\,dx-\frac{1}{4}(3\sinh(a))\int \frac{\cosh(bx^n}{x}\\ &= -\frac{3\mathbf{Chi}(bx^n)\sinh(a)}{4n}+\frac{\mathbf{Chi}(3bx^n)\sinh(3a)}{4n}-\frac{3\cosh(a)\mathbf{Shi}(bx^n)}{4n}+\frac{\cosh(3a)\mathbf{Shi}(3bx^n)}{4n}\end{aligned}$$

Mathematica [A] time = 0.0501692, size = 52, normalized size = 0.78

$$\frac{-3\sinh(a)\mathbf{Chi}(bx^n)+\sinh(3a)\mathbf{Chi}(3bx^n)-3\cosh(a)\mathbf{Shi}(bx^n)+\cosh(3a)\mathbf{Shi}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a] + CoshIntegral[3*b*x^n]*Sinh[3*a] - 3*Cosh[
a]*SinhIntegral[b*x^n] + Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n)
```

Maple [A] time = 0.109, size = 67, normalized size = 1.

$$\frac{e^{-3a}\mathrm{Ei}(1,3bx^n)}{8n}-\frac{3e^{-a}\mathrm{Ei}(1,bx^n)}{8n}-\frac{e^{3a}\mathrm{Ei}(1,-3bx^n)}{8n}+\frac{3e^{a}\mathrm{Ei}(1,-bx^n)}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^3/x,x)
```

```
[Out] 1/8/n*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a)*Ei(1,b*x^n)-1/8/n*exp(3*a)*Ei(1
,-3*b*x^n)+3/8/n*exp(a)*Ei(1,-b*x^n)
```

Maxima [A] time = 1.24995, size = 84, normalized size = 1.25

$$\frac{\mathrm{Ei}\left(3\,bx^{n}\right)e^{(3\,a)}}{8\,n}+\frac{3\,\mathrm{Ei}\left(-bx^{n}\right)e^{(-a)}}{8\,n}-\frac{\mathrm{Ei}\left(-3\,bx^{n}\right)e^{(-3\,a)}}{8\,n}-\frac{3\,\mathrm{Ei}\left(bx^{n}\right)e^{a}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x^n)*e^(3*a)/n + 3/8*Ei(-b*x^n)*e^(-a)/n - 1/8*Ei(-3*b*x^n)*e^(-
3*a)/n - 3/8*Ei(b*x^n)*e^a/n
```

Fricas [A] time = 1.87986, size = 374, normalized size = 5.58

$$(\cosh(3\,a)+\sinh(3\,a))\mathrm{Ei}\left(3\,b\cosh(n\log(x))+3\,b\sinh(n\log(x))\right)-3\,(\cosh(a)+\sinh(a))\mathrm{Ei}\left(b\cosh(n\log(x))+b\,\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/8*((cosh(3*a) + sinh(3*a))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) -
3*(cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*(cosh(a)
- sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) - (cosh(3*a) - sinh(3*a
))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{3}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x**n)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^n + a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^3/x, x)
```

3.73 $\int \frac{\sinh^3(a+bx^n)}{x^2} dx$

Optimal. Leaf size=154

$$-\frac{e^{3a}3^{\frac{1}{n}}(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^a(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-bx^n\right)}{8nx}-\frac{3e^{-a}(bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},bx^n\right)}{8nx}+\frac{e^{-3a}3^{\frac{1}{n}}(b$$

```
[Out] -(3^n^(-1)*E^(3*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n])/(8*n*x) + (3
*E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(8*n*x) - (3*(b*x^n)^n^(-1
)*Gamma[-n^(-1), b*x^n])/(8*E^a*n*x) + (3^n^(-1)*(b*x^n)^n^(-1)*Gamma[-n^(-
1), 3*b*x^n])/(8*E^(3*a)*n*x)
```

Rubi [A] time = 0.181779, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}3^{\frac{1}{n}}(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^a(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-bx^n\right)}{8nx}-\frac{3e^{-a}(bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},bx^n\right)}{8nx}+\frac{e^{-3a}3^{\frac{1}{n}}(b$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^3/x^2,x]
```


```
[Out] -(3^n^(-1)*E^(3*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n])/(8*n*x) + (3
*E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(8*n*x) - (3*(b*x^n)^n^(-1
)*Gamma[-n^(-1), b*x^n])/(8*E^a*n*x) + (3^n^(-1)*(b*x^n)^n^(-1)*Gamma[-n^(-
1), 3*b*x^n])/(8*E^(3*a)*n*x)
```


Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx &= \int \left(-\frac{3\sinh(a+bx^n)}{4x^2}+\frac{\sinh(3a+3bx^n)}{4x^2}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx^n)}{x^2}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^n)}{x^2}\,dx\\ &= -\left(\frac{1}{8}\int \frac{e^{-3a-3bx^n}}{x^2}\,dx\right)+\frac{1}{8}\int \frac{e^{3a+3bx^n}}{x^2}\,dx+\frac{3}{8}\int \frac{e^{-a-bx^n}}{x^2}\,dx-\frac{3}{8}\int \frac{e^{a+bx^n}}{x^2}\,dx\\ &= -\frac{3^{\frac{1}{n}}e^{3a}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^{a}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8nx}-\frac{3e^{-a}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},bx^n\right)}{8nx}+\frac{3^{\frac{1}{n}}e^{-3}}{}\end{aligned}$$

Mathematica [A] time = 1.33, size = 126, normalized size = 0.82

$$\frac{e^{-3a}\left(e^{6a}\left(-3^{\frac{1}{n}}\right)(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-3bx^n\right)+3e^{4a}(-bx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-bx^n\right)+(bx^n)^{\frac{1}{n}}\left(3^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},3bx^n\right)-\right.\right.}{8nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^3/x^2,x]
```

```
[Out] (-(3^n^(-1)*E^(6*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n]) + 3*E^(4*a)
*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)] + (b*x^n)^n^(-1)*(-3*E^(2*a)*Ga
mma[-n^(-1), b*x^n] + 3^n^(-1)*Gamma[-n^(-1), 3*b*x^n]))/(8*E^(3*a)*n*x)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{(\sinh(a+bx^n))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^3/x^2,x)
```

```
[Out] int(sinh(a+b*x^n)^3/x^2,x)
```

Maxima [A] time = 1.30212, size = 180, normalized size = 1.17

$$\frac{(3\,bx^n)^{\left(\frac{1}{n}\right)}e^{(-3\,a)}\Gamma\left(-\frac{1}{n},3\,bx^n\right)}{8\,nx}-\frac{3\,(bx^n)^{\left(\frac{1}{n}\right)}e^{(-a)}\Gamma\left(-\frac{1}{n},bx^n\right)}{8\,nx}+\frac{3\,(-bx^n)^{\left(\frac{1}{n}\right)}e^{a}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8\,nx}-\frac{(-3\,bx^n)^{\left(\frac{1}{n}\right)}e^{(3\,a)}\Gamma\left(-\frac{1}{n},-3\,bx\right.}{8\,nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^n)^(1/n)*e^(-3*a)*gamma(-1/n, 3*b*x^n)/(n*x) - 3/8*(b*x^n)^(1/n)
*e^(-a)*gamma(-1/n, b*x^n)/(n*x) + 3/8*(-b*x^n)^(1/n)*e^a*gamma(-1/n, -b*x^
n)/(n*x) - 1/8*(-3*b*x^n)^(1/n)*e^(3*a)*gamma(-1/n, -3*b*x^n)/(n*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sinh\left(bx^n+a\right)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^3\left(a+bx^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx^n + a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^3/x^2, x)
```

3.74 $\int (ex)^m (b\sinh(c+dx^n))^p\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((ex)^m (b\sinh(c+dx^n))^p, x\right)$$

```
[Out] Unintegrable[(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

Rubi [A] time = 0.02146, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m (b\sinh(c+dx^n))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (b\sinh(c+dx^n))^p\,dx = \int (ex)^m (b\sinh(c+dx^n))^p\,dx$$

Mathematica [A] time = 5.23899, size = 0, normalized size = 0.

$$\int (ex)^m (b\sinh(c+dx^n))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

Maple [A] time = 0.778, size = 0, normalized size = 0.

$$\int (ex)^m (b \sinh (c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(b*sinh(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m (b \sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m (b \sinh (dx^n + c))^p , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (c + dx^n))^p (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*sinh(c+d*x**n))**p,x)
```

```
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m (b\sinh(dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

3.75 $\int (ex)^m (a+b\sinh(c+dx^n))^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((ex)^m (a+b\sinh(c+dx^n))^p,x\right)$$

```
[Out] Unintegrable[(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

Rubi [A] time = 0.0250721, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m (a+b\sinh(c+dx^n))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a+b\sinh(c+dx^n))^p\,dx=\int (ex)^m (a+b\sinh(c+dx^n))^p\,dx$$

Mathematica [A] time = 8.06098, size = 0, normalized size = 0.

$$\int (ex)^m (a+b\sinh(c+dx^n))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

Maple [A] time = 0.661, size = 0, normalized size = 0.

$$\int (ex)^m (a + b \sinh (c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m (b \sinh (dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m (b \sinh (dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sinh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m (b \sinh (dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

3.76 $\int (ex)^{-1+n} (b\sinh(c+dx^n))^p \, dx$

Optimal. Leaf size=94

$$\frac{x^{-n}(ex)^n \cosh(c+dx^n)\,(b\sinh(c+dx^n))^{p+1}\; {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};-\sinh^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cosh^2(c+dx^n)}}$$

```
[Out] ((e*x)^n*Cosh[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, -Sinh
[c + d*x^n]^2]*(b*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cosh[
c + d*x^n]^2])
```

Rubi [A] time = 0.103774, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5322, 5320, 2643}

$$\frac{x^{-n}(ex)^n \cosh(c+dx^n)\,(b\sinh(c+dx^n))^{p+1}\; {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};-\sinh^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cosh^2(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*Cosh[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, -Sinh
[c + d*x^n]^2]*(b*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cosh[
c + d*x^n]^2])
```

Rule 5322

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
 + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
```

```
[(m + 1)/n], 0]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (ex)^{-1+n}(b\sinh(c+dx^n))^p\,dx &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(b\sinh(c+dx^n))^p\,dx}{e}\\ &= \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int (b\sinh(c+dx))^p\,dx,x,x^n\right)}{en}\\ &= \frac{x^{-n}(ex)^n\cosh(c+dx^n)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};-\sinh^2(c+dx^n)\right)(b\sinh(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cosh^2(c+dx^n)}}\end{aligned}$$

Mathematica [A] time = 0.138132, size = 93, normalized size = 0.99

$$-\frac{x^{-n}(ex)^n\sinh(2(c+dx^n))\left(-\sinh^2(c+dx^n)\right)^{\frac{1}{2}(-p-1)}(b\sinh(c+dx^n))^p\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3}{2};\cosh^2(dx^n+c)\right)}{2den}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] -((e*x)^n*Hypergeometric2F1[1/2, (1 - p)/2, 3/2, Cosh[c + d*x^n]^2]*(b*Sinh
[c + d*x^n])^p*(-Sinh[c + d*x^n]^2)^((-1 - p)/2)*Sinh[2*(c + d*x^n)])/(2*d*
e*n*x^n)
```

Maple [F] time = 0.838, size = 0, normalized size = 0.

$$\int (ex)^{-1+n}(b\sinh(c+dx^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1}\left(b\sinh\left(dx^n+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^{n-1}\left(b\sinh\left(dx^n+c\right)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(b*sinh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1} \left(b \sinh \left(dx^n + c\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

3.77 $\int (ex)^{-1+2n} (b\sinh(c+dx^n))^p \, dx$

Optimal. Leaf size=38

$$\frac{x^{-2n}(ex)^{2n}\text{Unintegrable}\left(x^{2n-1}(b\sinh(c+dx^n))^p, x\right)}{e}$$

```
[Out] ((e*x)^(2*n)*Unintegrable[x^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x])/(e*x^(2*n
))
```

Rubi [A] time = 0.0513811, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^{-1+2n} (b\sinh(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (b\sinh(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(b\sinh(c+dx^n))^p \, dx}{e}$$

Mathematica [A] time = 5.78723, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} (b\sinh(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x]
```

Maple [A] time = 0.753, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} (b \sinh (c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^{2n-1} (b \sinh (dx^n + c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(b*sinh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

3.78 $\int (ex)^{-1+n} (a + b\sinh(c + dx^n))^p \, dx$

Optimal. Leaf size=150

$$\frac{i\sqrt{2}x^{-n}(ex)^n \cosh(c+dx^n)\,(a+b\sinh(c+dx^n))^p \left(\frac{a+b\sinh(c+dx^n)}{a-ib}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-i\sinh(dx^n+c)),\frac{b(1-i\sinh(dx^n\cdots}{ia+b}\right.}{den\sqrt{1+i\sinh(c+dx^n)}}$$

```
[Out] (I*Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - I*Sinh[c + d*x^n])/2, (
b*(1 - I*Sinh[c + d*x^n]))/(I*a + b)]*Cosh[c + d*x^n]*(a + b*Sinh[c + d*x^n
])^p)/(d*e*n*x^n*Sqrt[1 + I*Sinh[c + d*x^n]]*((a + b*Sinh[c + d*x^n])/(a -
I*b))^p)
```

Rubi [A] time = 0.199267, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5322, 5320, 2665, 139, 138}

$$\frac{i\sqrt{2}x^{-n}(ex)^n \cosh(c+dx^n)\,(a+b\sinh(c+dx^n))^p \left(\frac{a+b\sinh(c+dx^n)}{a-ib}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-i\sinh(dx^n+c)),\frac{b(1-i\sinh(dx^n\cdots}{ia+b}\right.}{den\sqrt{1+i\sinh(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] (I*Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - I*Sinh[c + d*x^n])/2, (
b*(1 - I*Sinh[c + d*x^n]))/(I*a + b)]*Cosh[c + d*x^n]*(a + b*Sinh[c + d*x^n
])^p)/(d*e*n*x^n*Sqrt[1 + I*Sinh[c + d*x^n]]*((a + b*Sinh[c + d*x^n])/(a -
I*b))^p)
```

Rule 5322

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
 + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
```

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (ex)^{-1+n}\left(a+b\sinh\left(c+dx^n\right)\right)^p\,dx=\frac{(x^{-n}(ex)^n)\int x^{-1+n}\left(a+b\sinh\left(c+dx^n\right)\right)^p\,dx}{e}$$
$$=\frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (a+b\sinh(c+dx))^p\,dx,x,x^n\right)}{en}$$
$$=-\frac{(ix^{-n}(ex)^n\cosh\left(c+dx^n\right))\,\text{Subst}\left(\int \frac{(a-ibx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,i\sinh\left(c+dx^n\right)\right)}{den\sqrt{1-i\sinh\left(c+dx^n\right)}\sqrt{1+i\sinh\left(c+dx^n\right)}}$$
$$=-\frac{\left(ix^{-n}(ex)^n\cosh\left(c+dx^n\right)\left(a+b\sinh\left(c+dx^n\right)\right)^p\left(-\frac{a+b\sinh(c+dx^n)}{-a+ib}\right)^{-p}\right)\text{Subst}\Big(\int}{den\sqrt{1-i\sinh\left(c+dx^n\right)}\sqrt{1+i\sinh\left(c+dx\right.}}$$
$$=\frac{i\sqrt{2}x^{-n}(ex)^nF_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}\left(1-i\sinh\left(c+dx^n\right)\right),\frac{b(1-i\sinh(c+dx^n))}{ia+b}\right)\cosh\left(c+d\right.}{den\sqrt{1+i\sinh\left(c+dx^n\right)}}$$

Mathematica [A] time = 0.408769, size = 167, normalized size = 1.11

$$\frac{x^{-n}(ex)^n\text{sech}\left(c+dx^n\right)\sqrt{\frac{b(1-i\sinh(c+dx^n))}{b+ia}}\sqrt{\frac{b(1+i\sinh(c+dx^n))}{b-ia}}\left(a+b\sinh\left(c+dx^n\right)\right)^{p+1}F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{a+b\sinh(dx^n+c)}{a+ib},\right.}{bden(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sinh[c + d*x^n])/(a + I*b)
, (a + b*Sinh[c + d*x^n])/(a - I*b)]*Sech[c + d*x^n]*Sqrt[(b*(1 - I*Sinh[c
+ d*x^n]))/(I*a + b)]*Sqrt[(b*(1 + I*Sinh[c + d*x^n]))/((-I)*a + b)]*(a + b
*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Maple [F] time = 0.701, size = 0, normalized size = 0.

$$\int (ex)^{-1+n}\left(a+b\sinh\left(c+dx^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1}(b\sinh(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^{n-1}(b\sinh(dx^n+c)+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*sinh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1}(b\sinh(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

3.79 $\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$

Optimal. Leaf size=40

$$\frac{x^{-2n}(ex)^{2n}\text{Unintegrable}\left(x^{2n-1}(a+b\sinh(c+dx^n))^p,x\right)}{e}$$

```
[Out] ((e*x)^(2*n)*Unintegrable[x^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x])/(e*x^
(2*n))
```

Rubi [A] time = 0.0595259, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x])/(e*x^(2
*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx}{e}$$

Mathematica [A] time = 8.35159, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x]
```

Maple [A] time = 0.674, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} (a + b\sinh(c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b\sinh(dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^{2n-1} (b\sinh(dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sinh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sinh (dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

3.80 $\int (ex)^m \sinh^3(a + bx^n) \, dx$

Optimal. Leaf size=220

$$-\frac{e^{3a}3^{-\frac{m+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-3bx^n\right)}{8en}+\frac{3e^a(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{8en}-\frac{3e^{-a}(ex)^{m+1}(bx}{}$$

```
[Out] -(E^(3*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(8*3^((1 + m)/n)*e*n*(-
(b*x^n))^((1 + m)/n)) + (3*E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(8
*e*n*(-(b*x^n))^((1 + m)/n)) - (3*(e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(8
*e*E^a*n*(b*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])/(
8*3^((1 + m)/n)*e*E^(3*a)*n*(b*x^n)^((1 + m)/n))
```

Rubi [A] time = 0.234303, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}3^{-\frac{m+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-3bx^n\right)}{8en}+\frac{3e^a(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{8en}-\frac{3e^{-a}(ex)^{m+1}(bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^n]^3,x]
```


```
[Out] -(E^(3*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(8*3^((1 + m)/n)*e*n*(-
(b*x^n))^((1 + m)/n)) + (3*E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(8
*e*n*(-(b*x^n))^((1 + m)/n)) - (3*(e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(8
*e*E^a*n*(b*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])/(
8*3^((1 + m)/n)*e*E^(3*a)*n*(b*x^n)^((1 + m)/n))
```


Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
```

```
x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (ex)^m \sinh^3(a+bx^n)\,dx = \int \left(-\frac{3}{4}(ex)^m \sinh(a+bx^n)+\frac{1}{4}(ex)^m \sinh(3a+3bx^n)\right)dx$$
$$= \frac{1}{4}\int (ex)^m \sinh(3a+3bx^n)\,dx - \frac{3}{4}\int (ex)^m \sinh(a+bx^n)\,dx$$
$$= -\left(\frac{1}{8}\int e^{-3a-3bx^n}(ex)^m\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}(ex)^m\,dx+\frac{3}{8}\int e^{-a-bx^n}(ex)^m\,dx-\frac{3}{8}\int e^{a+bx^n}(ex)^r$$
$$= -\frac{3^{-\frac{1+m}{n}}e^{3a}(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)}{8en}+\frac{3e^{a}(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)}{8en}-\frac{3e}{}$$

Mathematica [A] time = 2.0467, size = 185, normalized size = 0.84

$$\frac{e^{-3a}x3^{-\frac{m+1}{n}}(ex)^m\left(-b^2x^{2n}\right)^{-\frac{m+1}{n}}\left((-bx^n)^{\frac{m+1}{n}}\left(\text{Gamma}\left(\frac{m+1}{n},3bx^n\right)-e^{2a}3^{\frac{m+n+1}{n}}\text{Gamma}\left(\frac{m+1}{n},bx^n\right)\right)-e^{6a}(bx^n)^{\frac{m+1}{n}}\text{Gamm}\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n]^3,x]
```

```
[Out] (x*(e*x)^m*(-(E^(6*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -3*b*x^n]) + 3^(
(1 + m + n)/n)*E^(4*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)] + (-(
b*x^n))^((1 + m)/n)*(-(3^((1 + m + n)/n)*E^(2*a)*Gamma[(1 + m)/n, b*x^n]) +
 Gamma[(1 + m)/n, 3*b*x^n])))/(8*3^((1 + m)/n)*E^(3*a)*n*(-(b^2*x^(2*n)))^(
(1 + m)/n))
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int (ex)^m (\sinh(a+bx^n))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b*x^n)^3,x)
```

```
[Out] int((e*x)^m*sinh(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh(bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh(bx^n + a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^3(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b*x**n)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**n)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh \left(bx^n + a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a)^3, x)
```

3.81 $\int (ex)^m \sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=143

$$-\frac{e^{2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2bx^n\right)}{en}-\frac{e^{-2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2bx^n\right)}{en}-\frac{(ex)}{2e(m}$$

```
[Out] -(e*x)^(1 + m)/(2*e*(1 + m)) - (E^(2*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -2*b
*x^n])/(2^((1 + m + 2*n)/n)*e*n*(-(b*x^n))^((1 + m)/n)) - ((e*x)^(1 + m)*Ga
mma[(1 + m)/n, 2*b*x^n])/(2^((1 + m + 2*n)/n)*e*E^(2*a)*n*(b*x^n)^((1 + m)/
n))
```

Rubi [A] time = 0.174381, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2bx^n\right)}{en}-\frac{e^{-2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2bx^n\right)}{en}-\frac{(ex)}{2e(m}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^n]^2,x]
```

```
[Out] -(e*x)^(1 + m)/(2*e*(1 + m)) - (E^(2*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -2*b
*x^n])/(2^((1 + m + 2*n)/n)*e*n*(-(b*x^n))^((1 + m)/n)) - ((e*x)^(1 + m)*Ga
mma[(1 + m)/n, 2*b*x^n])/(2^((1 + m + 2*n)/n)*e*E^(2*a)*n*(b*x^n)^((1 + m)/
n))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^2(a+bx^n)\,dx &= \int \left(-\frac{1}{2}(ex)^m + \frac{1}{2}(ex)^m \cosh(2a+2bx^n)\right) dx \\ &= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{2}\int (ex)^m \cosh(2a+2bx^n)\,dx \\ &= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{4}\int e^{-2a-2bx^n}(ex)^m\,dx + \frac{1}{4}\int e^{2a+2bx^n}(ex)^m\,dx \\ &= -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a}(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2bx^n\right)}{en} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a}(ex)^{1+m}\left(bx^n\right)^{-}}{en}\end{aligned}$$

Mathematica [A] time = 1.90741, size = 117, normalized size = 0.82

$$-\frac{x(ex)^m\left(e^{2a}(m+1)2^{-\frac{m+1}{n}}\left(-bx^n\right)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2bx^n\right)+e^{-2a}(m+1)2^{-\frac{m+1}{n}}\left(bx^n\right)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2bx^n\right)+2n\right)}{4(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n]^2,x]
```

```
[Out] -(x*(e*x)^m*(2*n + (E^(2*a)*(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^((1 + m)
/n)*(-(b*x^n))^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, 2*b*x^n])/(2^((1 +
m)/n)*E^(2*a)*(b*x^n)^((1 + m)/n))))/(4*(1 + m)*n)
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int (ex)^m (\sinh(a+bx^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b*x^n)^2,x)
```

```
[Out] int((e*x)^m*sinh(a+b*x^n)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh(bx^n + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh^2(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh (bx^n + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a)^2, x)
```

3.82 $\int (ex)^m \sinh(a+bx^n)\,dx$

Optimal. Leaf size=99

$$\frac{e^{-a}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\operatorname{Gamma}\left(\frac{m+1}{n},bx^n\right)}{2en}-\frac{e^{a}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\operatorname{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{2en}$$

```
[Out] -(E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(2*e*n*(-(b*x^n))^((1 + m)/
n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(2*e*E^a*n*(b*x^n)^((1 + m)/n
))
```

Rubi [A] time = 0.0705051, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5360, 2218}

$$\frac{e^{-a}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\operatorname{Gamma}\left(\frac{m+1}{n},bx^n\right)}{2en}-\frac{e^{a}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\operatorname{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{2en}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(2*e*n*(-(b*x^n))^((1 + m)/
n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(2*e*E^a*n*(b*x^n)^((1 + m)/n
))
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (ex)^m \sinh\left(a+bx^n\right)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}(ex)^m\,dx\right)+\frac{1}{2}\int e^{a+bx^n}(ex)^m\,dx$$
$$= -\frac{e^a(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)}{2en}+\frac{e^{-a}(ex)^{1+m}\left(bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)}{2en}$$

Mathematica [A] time = 0.170415, size = 102, normalized size = 1.03

$$-\frac{x(ex)^m\left(-b^2x^{2n}\right)^{-\frac{m+1}{n}}\left((\sinh(a)+\cosh(a))\left(bx^n\right)^{\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)-(\cosh(a)-\sinh(a))\left(-bx^n\right)^{\frac{m+1}{n}}\text{Gamma}\left(\frac{m\text{-}}{n}\right.\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n],x]
```

```
[Out] -(x*(e*x)^m*(-((-(b*x^n))^((1 + m)/n)*Gamma[(1 + m)/n, b*x^n]*(Cosh[a] - Si
nh[a])) + (b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)]*(Cosh[a] + Sinh[a]
)))/(2*n*(-(b^2*x^(2*n)))^((1 + m)/n))
```

Maple [C] time = 0.22, size = 115, normalized size = 1.2

$$\frac{(ex)^m x\sinh(a)}{1+m}{}_1F_2\left(\frac{m}{2n}+\frac{1}{2n};\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n};\frac{x^{2n}b^2}{4}\right)+\frac{(ex)^m x^{n+1}b\cosh(a)}{m+n+1}{}_1F_2\left(\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n};\frac{x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b*x^n),x)
```

```
[Out] (e*x)^m/(1+m)*x*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],1/4*x^(2*n)
*b^2)*sinh(a)+(e*x)^m/(m+n+1)*x^(n+1)*b*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,
3/2+1/2/n*m+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sinh\left(bx^n + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b*x**n),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sinh\left(bx^n + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a), x)
```

3.83 $\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx$

Optimal. Leaf size=27

$$x^{-m}(ex)^m \text{Unintegrable}\left(x^m \operatorname{csch}^2(a+bx^n),x\right)$$

```
[Out] ((e*x)^m*Unintegrable[x^m*Csch[a + b*x^n]^2, x])/x^m
```

Rubi [A] time = 0.0377175, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b*x^n]^2,x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b*x^n]^2, x])/x^m
```

Rubi steps

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx = \left(x^{-m}(ex)^m\right)\int x^m \operatorname{csch}^2(a+bx^n)\,dx$$

Mathematica [A] time = 24.3329, size = 0, normalized size = 0.

$$\int (ex)^m \operatorname{csch}^2(a+bx^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b*x^n]^2,x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b*x^n]^2, x]
```

Maple [A] time = 0.13, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(\sinh(a+bx^n))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a+b*x^n)^2,x)
```

```
[Out] int((e*x)^m/sinh(a+b*x^n)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4\,e^m(m-n+1)\int \frac{x^m}{4\left(bnx^n + bne^{(bx^n+n\log(x)+a)}\right)}\,dx + 4\,e^m(m-n+1)\int -\frac{x^m}{4\left(bnx^n - bne^{(bx^n+n\log(x)+a)}\right)}\,dx + \frac{2}{bnx^n - bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -4*e^m*(m - n + 1)*integrate(1/4*x^m/(b*n*x^n + b*n*e^(b*x^n + n*log(x) + a
)), x) + 4*e^m*(m - n + 1)*integrate(-1/4*x^m/(b*n*x^n - b*n*e^(b*x^n + n*l
og(x) + a)), x) + 2*e^m*x*x^m/(b*n*x^n - b*n*e^(2*b*x^n + n*log(x) + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex)^m}{\sinh(bx^n+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/sinh(b*x^n + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh^2(a + bx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(a+b*x**n)**2,x)
```

```
[Out] Integral((e*x)**m/sinh(a + b*x**n)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sinh(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(b*x^n + a)^2, x)
```

3.84 $\int x^{-1-n}\sinh(a+bx^n)\,dx$

Optimal. Leaf size=45

$$\frac{b\cosh(a)\mathrm{Chi}(bx^n)}{n}+\frac{b\sinh(a)\mathrm{Shi}(bx^n)}{n}-\frac{x^{-n}\sinh(a+bx^n)}{n}$$

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^n])/n - Sinh[a + b*x^n]/(n*x^n) + (b*Sinh[a]*Si
nhIntegral[b*x^n])/n
```

Rubi [A] time = 0.0928278, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5320, 3297, 3303, 3298, 3301}

$$\frac{b\cosh(a)\mathrm{Chi}(bx^n)}{n}+\frac{b\sinh(a)\mathrm{Shi}(bx^n)}{n}-\frac{x^{-n}\sinh(a+bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n],x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^n])/n - Sinh[a + b*x^n]/(n*x^n) + (b*Sinh[a]*Si
nhIntegral[b*x^n])/n
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\sinh\left(a+bx^n\right)dx &= \frac{\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-n}\sinh\left(a+bx^n\right)}{n}+\frac{b\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-n}\sinh\left(a+bx^n\right)}{n}+\frac{(b\cosh(a))\,\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,x^n\right)}{n}+\frac{(b\sinh(a))\,\text{Subst}\left(\int \frac{\sinh(b}{x}\right.}{n}\\ &= \frac{b\cosh(a)\textbf{Chi}\left(bx^n\right)}{n}-\frac{x^{-n}\sinh\left(a+bx^n\right)}{n}+\frac{b\sinh(a)\textbf{Shi}\left(bx^n\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0593288, size = 46, normalized size = 1.02

$$\frac{x^{-n}\left(b\cosh(a)x^n\textbf{Chi}\left(bx^n\right)+b\sinh(a)x^n\textbf{Shi}\left(bx^n\right)-\sinh\left(a+bx^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n],x]
```

```
[Out] (b*x^n*Cosh[a]*CoshIntegral[b*x^n] - Sinh[a + b*x^n] + b*x^n*Sinh[a]*SinhIn
tegral[b*x^n])/(n*x^n)
```

Maple [A] time = 0.036, size = 74, normalized size = 1.6

$$\frac{\mathrm{e}^{-a-bx^n}}{2\,nx^n}-\frac{b\mathrm{e}^{-a}\mathrm{Ei}\,(1,bx^n)}{2\,n}-\frac{\mathrm{e}^{a+bx^n}}{2\,nx^n}-\frac{\mathrm{e}^{a}b\mathrm{Ei}\,(1,-bx^n)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sinh(a+b*x^n),x)
```

```
[Out] 1/2/n*exp(-a-b*x^n)/(x^n)-1/2/n*b*exp(-a)*Ei(1,b*x^n)-1/2*exp(a+b*x^n)/(x^n
)/n-1/2/n*b*exp(a)*Ei(1,-b*x^n)
```

Maxima [A] time = 1.33458, size = 46, normalized size = 1.02

$$\frac{be^{(-a)}\Gamma\,(-1,bx^n)}{2\,n}+\frac{be^{a}\Gamma\,(-1,-bx^n)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-a)*gamma(-1, b*x^n)/n + 1/2*b*e^a*gamma(-1, -b*x^n)/n
```

Fricas [B] time = 1.89034, size = 462, normalized size = 10.27

$$((b\cosh(a)+b\sinh(a))\cosh(n\log(x))+(b\cosh(a)+b\sinh(a))\sinh(n\log(x)))\mathrm{Ei}\,(b\cosh(n\log(x))+b\sinh(n\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh
(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + ((b*cosh(a) - b*sinh(
a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*l
og(x)) - b*sinh(n*log(x))) - 2*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a
))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1} \sinh\left(bx^n + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a), x)
```

3.85 $\int x^{-1-n} \sinh^2(a + bx^n)\, dx$

Optimal. Leaf size=67

$$\frac{b \sinh(2a)\text{Chi}(2bx^n)}{n} + \frac{b \cosh(2a)\text{Shi}(2bx^n)}{n} - \frac{x^{-n}\cosh(2(a + bx^n))}{2n} + \frac{x^{-n}}{2n}$$

```
[Out] 1/(2*n*x^n) - Cosh[2*(a + b*x^n)]/(2*n*x^n) + (b*CoshIntegral[2*b*x^n]*Sinh
[2*a])/n + (b*Cosh[2*a]*SinhIntegral[2*b*x^n])/n
```

Rubi [A] time = 0.122997, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5362, 5321, 3297, 3303, 3298, 3301}

$$\frac{b \sinh(2a)\text{Chi}(2bx^n)}{n} + \frac{b \cosh(2a)\text{Shi}(2bx^n)}{n} - \frac{x^{-n}\cosh(2(a + bx^n))}{2n} + \frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n]^2,x]
```

```
[Out] 1/(2*n*x^n) - Cosh[2*(a + b*x^n)]/(2*n*x^n) + (b*CoshIntegral[2*b*x^n]*Sinh
[2*a])/n + (b*Cosh[2*a]*SinhIntegral[2*b*x^n])/n
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5321

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1-n}\sinh^2(a+bx^n)\,dx &= \int\left(-\frac{1}{2}x^{-1-n}+\frac{1}{2}x^{-1-n}\cosh(2a+2bx^n)\right)dx\\
&= \frac{x^{-n}}{2n}+\frac{1}{2}\int x^{-1-n}\cosh(2a+2bx^n)\,dx\\
&= \frac{x^{-n}}{2n}+\frac{\text{Subst}\left(\int\frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}\\
&= \frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{b\,\text{Subst}\left(\int\frac{\sinh(2a+2bx)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{(b\cosh(2a))\,\text{Subst}\left(\int\frac{\sinh(2bx)}{x}\,dx,x,x^n\right)}{n}+\frac{(b\sinh(2a))\,\text{Su}}{}\\
&= \frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{b\,\text{Chi}(2bx^n)\sinh(2a)}{n}+\frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.130863, size = 54, normalized size = 0.81

$$\frac{x^{-n}\left(b\sinh(2a)x^n\text{Chi}\left(2bx^n\right)+b\cosh(2a)x^n\text{Shi}\left(2bx^n\right)-\sinh^2\left(a+bx^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n]^2,x]
```

```
[Out] (b*x^n*CoshIntegral[2*b*x^n]*Sinh[2*a] - Sinh[a + b*x^n]^2 + b*x^n*Cosh[2*a
]*SinhIntegral[2*b*x^n])/(n*x^n)
```

Maple [A] time = 0.073, size = 90, normalized size = 1.3

$$\frac{1}{2\,nx^n}-\frac{e^{-2\,a-2\,bx^n}}{4\,nx^n}+\frac{be^{-2\,a}\text{Ei}\left(1,2\,bx^n\right)}{2\,n}-\frac{e^{2\,a+2\,bx^n}}{4\,nx^n}-\frac{be^{2\,a}\text{Ei}\left(1,-2\,bx^n\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sinh(a+b*x^n)^2,x)
```

```
[Out] 1/2/n/(x^n)-1/4/n*exp(-2*a-2*b*x^n)/(x^n)+1/2/n*b*exp(-2*a)*Ei(1,2*b*x^n)-1
/4/(x^n)*exp(2*a+2*b*x^n)/n-1/2/n*b*exp(2*a)*Ei(1,-2*b*x^n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89354, size = 567, normalized size = 8.46

$$\frac{\left(\left(b\cosh\left(2\,a\right)+b\sinh\left(2\,a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(2\,a\right)+b\sinh\left(2\,a\right)\right)\sinh\left(n\log\left(x\right)\right)\right)\text{Ei}\left(2\,b\cosh\left(n\log\left(x\right)\right)+2\,b\,s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(2*a) + b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) + b*sinh(2*
a))*sinh(n*log(x)))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) - ((b*cosh(
2*a) - b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) - b*sinh(2*a))*sinh(n*log
(x)))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - cosh(b*cosh(n*log(x))
+ b*sinh(n*log(x)) + a)^2 - sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2
 + 1)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1} \sinh(bx^n + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a)^2, x)
```

3.86 $\int x^{-1-n}\sinh^3\left(a+bx^n\right)\,dx$

Optimal. Leaf size=113

$$-\frac{3b\cosh(a)\text{Chi}\left(bx^n\right)}{4n}+\frac{3b\cosh(3a)\text{Chi}\left(3bx^n\right)}{4n}-\frac{3b\sinh(a)\text{Shi}\left(bx^n\right)}{4n}+\frac{3b\sinh(3a)\text{Shi}\left(3bx^n\right)}{4n}+\frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n}$$

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*CoshIntegral[3*b*
x^n])/(4*n) + (3*Sinh[a + b*x^n])/(4*n*x^n) - Sinh[3*(a + b*x^n)]/(4*n*x^n)
 - (3*b*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Sinh[3*a]*SinhIntegral[3*
b*x^n])/(4*n)
```

Rubi [A] time = 0.216987, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5362, 5320, 3297, 3303, 3298, 3301}

$$-\frac{3b\cosh(a)\text{Chi}\left(bx^n\right)}{4n}+\frac{3b\cosh(3a)\text{Chi}\left(3bx^n\right)}{4n}-\frac{3b\sinh(a)\text{Shi}\left(bx^n\right)}{4n}+\frac{3b\sinh(3a)\text{Shi}\left(3bx^n\right)}{4n}+\frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n]^3,x]
```

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*CoshIntegral[3*b*
x^n])/(4*n) + (3*Sinh[a + b*x^n])/(4*n*x^n) - Sinh[3*(a + b*x^n)]/(4*n*x^n)
 - (3*b*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Sinh[3*a]*SinhIntegral[3*
b*x^n])/(4*n)
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\sinh^3(a+bx^n)\,dx &= \int \left(-\frac{3}{4}x^{-1-n}\sinh(a+bx^n)+\frac{1}{4}x^{-1-n}\sinh(3a+3bx^n)\right)dx\\
&= \frac{1}{4}\int x^{-1-n}\sinh(3a+3bx^n)\,dx-\frac{3}{4}\int x^{-1-n}\sinh(a+bx^n)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(3a+3bx)}{x^2}\,dx,x,x^n\right)}{4n}-\frac{3\,\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,x^n\right)}{4n}\\
&= \frac{3x^{-n}\sinh(a+bx^n)}{4n}-\frac{x^{-n}\sinh(3(a+bx^n))}{4n}-\frac{(3b)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,x^n\right)}{4n}+\frac{(3b)\,\text{S}
\\
&= \frac{3x^{-n}\sinh(a+bx^n)}{4n}-\frac{x^{-n}\sinh(3(a+bx^n))}{4n}-\frac{(3b\cosh(a))\,\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,x^n\right)}{4n}+
\\
&= -\frac{3b\cosh(a)\mathbf{Chi}(bx^n)}{4n}+\frac{3b\cosh(3a)\mathbf{Chi}(3bx^n)}{4n}+\frac{3x^{-n}\sinh(a+bx^n)}{4n}-\frac{x^{-n}\sinh(3(a+
\end{aligned}$$

Mathematica [A] time = 0.200762, size = 95, normalized size = 0.84

$$-\frac{x^{-n}\left(3b\cosh(a)x^n\mathrm{Chi}\left(bx^n\right)-3b\cosh(3a)x^n\mathrm{Chi}\left(3bx^n\right)+3b\sinh(a)x^n\mathrm{Shi}\left(bx^n\right)-3b\sinh(3a)x^n\mathrm{Shi}\left(3bx^n\right)-3\sinh(a\right.}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(3*b*x^n*Cosh[a]*CoshIntegral[b*x^n] - 3*b*x^n*Cosh[3*a]*CoshIntegral[3*b*
x^n] - 3*Sinh[a + b*x^n] + Sinh[3*(a + b*x^n)] + 3*b*x^n*Sinh[a]*SinhIntegr
al[b*x^n] - 3*b*x^n*Sinh[3*a]*SinhIntegral[3*b*x^n])/(4*n*x^n)
```

Maple [A] time = 0.087, size = 152, normalized size = 1.4

$$\frac{e^{-3a-3bx^n}}{8nx^n}-\frac{3be^{-3a}\mathrm{Ei}\left(1,3bx^n\right)}{8n}-\frac{3e^{-a-bx^n}}{8nx^n}+\frac{3be^{-a}\mathrm{Ei}\left(1,bx^n\right)}{8n}-\frac{e^{3a+3bx^n}}{8nx^n}-\frac{3be^{3a}\mathrm{Ei}\left(1,-3bx^n\right)}{8n}+\frac{3e^{a+bx^n}}{8nx^n}+\frac{3e^{a}b\mathrm{Ei}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sinh(a+b*x^n)^3,x)
```

```
[Out] 1/8/n*exp(-3*a-3*b*x^n)/(x^n)-3/8/n*b*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a-
b*x^n)/(x^n)+3/8/n*b*exp(-a)*Ei(1,b*x^n)-1/8/(x^n)*exp(3*a+3*b*x^n)/n-3/8/n
*b*exp(3*a)*Ei(1,-3*b*x^n)+3/8*exp(a+b*x^n)/(x^n)/n+3/8/n*b*exp(a)*Ei(1,-b*
x^n)
```

Maxima [A] time = 1.3355, size = 95, normalized size = 0.84

$$\frac{3be^{(-3a)}\Gamma\left(-1,3bx^n\right)}{8n}-\frac{3be^{(-a)}\Gamma\left(-1,bx^n\right)}{8n}-\frac{3be^{a}\Gamma\left(-1,-bx^n\right)}{8n}+\frac{3be^{(3a)}\Gamma\left(-1,-3bx^n\right)}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 3/8*b*e^(-3*a)*gamma(-1, 3*b*x^n)/n - 3/8*b*e^(-a)*gamma(-1, b*x^n)/n - 3/8
*b*e^a*gamma(-1, -b*x^n)/n + 3/8*b*e^(3*a)*gamma(-1, -3*b*x^n)/n
```

Fricas [B] time = 1.89298, size = 986, normalized size = 8.73

$$-\frac{2\,\sinh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^{3}-3\left(\left(b\cosh\left(3\,a\right)+b\sinh\left(3\,a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(3\,a\right)+b\,\mathrm{s}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^3 - 3*((b*cosh(3*a) +
 b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) + b*sinh(3*a))*sinh(n*log(x)))*
Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) + 3*((b*cosh(a) + b*sinh(a))*co
sh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*log(x)))*Ei(b*cosh(n*log(x))
+ b*sinh(n*log(x))) + 3*((b*cosh(a) - b*sinh(a))*cosh(n*log(x)) + (b*cosh(a
) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) - 3
*((b*cosh(3*a) - b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) - b*sinh(3*a))*
sinh(n*log(x)))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))) + 6*(cosh(b*co
sh(n*log(x)) + b*sinh(n*log(x)) + a)^2 - 1)*sinh(b*cosh(n*log(x)) + b*sinh(
n*log(x)) + a))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1}\sinh\left(bx^{n}+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a)^3, x)
```

3.87 $\int x^{-1+\frac{n}{2}} \sinh(a+bx^n)\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\pi}e^a\text{Erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}-\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[b]*x^(n/2)])/(2*Sqrt[b]*E^a*n) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[b]*x^(n/2)])/(2*Sqrt[b]*n)
```

Rubi [A] time = 0.0447823, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5356, 5298, 2204, 2205}

$$\frac{\sqrt{\pi}e^a\text{Erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}-\frac{\sqrt{\pi}e^{-a}\text{Erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)*Sinh[a + b*x^n],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[b]*x^(n/2)])/(2*Sqrt[b]*E^a*n) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[b]*x^(n/2)])/(2*Sqrt[b]*n)
```

Rule 5356

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/(m + 1), Subst[Int[(a + b*Sinh[c + d*x^Simplify[n/(m + 1)]])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegerQ[p] && N
eQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] &&  !IntegerQ[n]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^{-1+\frac{n}{2}}\sinh(a+bx^n)\,dx &= \frac{2\,\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx,x,x^{n/2}\right)}{n}\\ &= -\frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx,x,x^{n/2}\right)}{n}+\frac{\text{Subst}\left(\int e^{a+bx^2}\,dx,x,x^{n/2}\right)}{n}\\ &= -\frac{e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}+\frac{e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}\end{aligned}$$

Mathematica [A] time = 1.50246, size = 60, normalized size = 0.85

$$\frac{\sqrt{\pi}\left((\sinh(a)-\cosh(a))\text{Erf}\left(\sqrt{b}x^{n/2}\right)+(\sinh(a)+\cosh(a))\text{Erfi}\left(\sqrt{b}x^{n/2}\right)\right)}{2\sqrt{b}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)*Sinh[a + b*x^n],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x^(n/2)]*(-Cosh[a] + Sinh[a]) + Erfi[Sqrt[b]*x^(n/2)
]*(Cosh[a] + Sinh[a])))/(2*Sqrt[b]*n)
```

Maple [A] time = 0.057, size = 54, normalized size = 0.8

$$-\frac{e^{-a}\sqrt{\pi}}{2n}\text{Erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)\frac{1}{\sqrt{b}}+\frac{e^{a}\sqrt{\pi}}{2n}\text{Erf}\left(\sqrt{-b}x^{\frac{n}{2}}\right)\frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)*sinh(a+b*x^n),x)
```

```
[Out] -1/2/n*exp(-a)*Pi^(1/2)/b^(1/2)*erf(x^(1/2*n)*b^(1/2))+1/2/n*exp(a)*Pi^(1/2
)/(-b)^(1/2)*erf((-b)^(1/2)*x^(1/2*n))
```

Maxima [A] time = 1.3147, size = 93, normalized size = 1.31

$$-\frac{\sqrt{\pi}x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{bx^n}\right)-1\right)e^{(-a)}}{2\sqrt{bx^n}n}+\frac{\sqrt{\pi}x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{-bx^n}\right)-1\right)e^{a}}{2\sqrt{-bx^n}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(b*x^n)) - 1)*e^(-a)/(sqrt(b*x^n)*n) + 1/2
*sqrt(pi)*x^(1/2*n)*(erf(sqrt(-b*x^n)) - 1)*e^a/(sqrt(-b*x^n)*n)
```

Fricas [A] time = 1.95006, size = 333, normalized size = 4.69

$$-\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\operatorname{erf}\left(\sqrt{-b}x\cosh\left(\frac{1}{2}(n-2)\log(x)\right)+\sqrt{-b}x\sinh\left(\frac{1}{2}(n-2)\log(x)\right)\right)+\sqrt{\pi}\sqrt{b}(\cosh(a)-\mathrm{si}}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x*cosh(1/2*(n - 2)
*log(x)) + sqrt(-b)*x*sinh(1/2*(n - 2)*log(x))) + sqrt(pi)*sqrt(b)*(cosh(a)
 - sinh(a))*erf(sqrt(b)*x*cosh(1/2*(n - 2)*log(x)) + sqrt(b)*x*sinh(1/2*(n
- 2)*log(x))))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{n}{2}-1}\sinh\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)*sinh(a+b*x**n),x)
```

```
[Out] Integral(x**(n/2 - 1)*sinh(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{1}{2}n-1}\sinh(bx^n+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*n - 1)*sinh(b*x^n + a), x)
```

3.88 $\int x^2 \sinh\left((a+bx)^2\right) dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{\pi}a^2\text{Erf}(a+bx)}{4b^3}+\frac{\sqrt{\pi}a^2\text{Erfi}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\text{Erf}(a+bx)}{8b^3}-\frac{\sqrt{\pi}\text{Erfi}(a+bx)}{8b^3}-\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left((a+\right.}{2b^3}$$

```
[Out] -((a*Cosh[(a + b*x)^2])/b^3) + ((a + b*x)*Cosh[(a + b*x)^2])/(2*b^3) - (Sqr
t[Pi]*Erf[a + b*x])/(8*b^3) - (a^2*Sqrt[Pi]*Erf[a + b*x])/(4*b^3) - (Sqrt[P
i]*Erfi[a + b*x])/(8*b^3) + (a^2*Sqrt[Pi]*Erfi[a + b*x])/(4*b^3)
```

Rubi [A] time = 0.0961929, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5364, 6742, 5298, 2204, 2205, 5320, 2638, 5324, 5299}

$$-\frac{\sqrt{\pi}a^2\text{Erf}(a+bx)}{4b^3}+\frac{\sqrt{\pi}a^2\text{Erfi}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\text{Erf}(a+bx)}{8b^3}-\frac{\sqrt{\pi}\text{Erfi}(a+bx)}{8b^3}-\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left((a+\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[(a + b*x)^2],x]
```

```
[Out] -((a*Cosh[(a + b*x)^2])/b^3) + ((a + b*x)*Cosh[(a + b*x)^2])/(2*b^3) - (Sqr
t[Pi]*Erf[a + b*x])/(8*b^3) - (a^2*Sqrt[Pi]*Erf[a + b*x])/(4*b^3) - (Sqrt[P
i]*Erfi[a + b*x])/(8*b^3) + (a^2*Sqrt[Pi]*Erfi[a + b*x])/(4*b^3)
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int x^2 \sinh\left((a+bx)^2\right) dx = \frac{\text{Subst}\left(\int (-a+x)^2 \sinh\left(x^2\right) dx, x, a+bx\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(a^2 \sinh\left(x^2\right) - 2ax \sinh\left(x^2\right) + x^2 \sinh\left(x^2\right)\right) dx, x, a+bx\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int x^2 \sinh\left(x^2\right) dx, x, a+bx\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int x \sinh\left(x^2\right) dx, x, a+bx\right)}{b^3} + \frac{a^2\,\text{Subst}\left(\right.}{}$$
$$= \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3} - \frac{\text{Subst}\left(\int \cosh\left(x^2\right) dx, x, a+bx\right)}{2b^3} - \frac{a\,\text{Subst}\left(\int \sinh(x)\, dx, x, (a\right.}{b^3}$$
$$= -\frac{a\cosh\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3} - \frac{a^2\sqrt{\pi}\,\text{erf}(a+bx)}{4b^3} + \frac{a^2\sqrt{\pi}\,\text{erfi}(a+bx)}{4b^3} - \frac{\text{S}}{}$$
$$= -\frac{a\cosh\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3} - \frac{\sqrt{\pi}\,\text{erf}(a+bx)}{8b^3} - \frac{a^2\sqrt{\pi}\,\text{erf}(a+bx)}{4b^3} - \frac{\sqrt{\pi}}{}$$

Mathematica [A] time = 0.128499, size = 63, normalized size = 0.56

$$\frac{-\sqrt{\pi}\left(2a^2+1\right)\text{Erf}(a+bx) + \sqrt{\pi}\left(2a^2-1\right)\text{Erfi}(a+bx) - 4(a-bx)\cosh\left((a+bx)^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[(a + b*x)^2],x]
```

```
[Out] (-4*(a - b*x)*Cosh[(a + b*x)^2] - (1 + 2*a^2)*Sqrt[Pi]*Erf[a + b*x] + (-1 +
 2*a^2)*Sqrt[Pi]*Erfi[a + b*x])/(8*b^3)
```

Maple [C] time = 0.049, size = 136, normalized size = 1.2

$$\frac{x\mathrm{e}^{-(bx+a)^2}}{4\,b^2} - \frac{a\mathrm{e}^{-(bx+a)^2}}{4\,b^3} - \frac{a^2\text{Erf}\,(bx+a)\,\sqrt{\pi}}{4\,b^3} - \frac{\text{Erf}\,(bx+a)\,\sqrt{\pi}}{8\,b^3} + \frac{x\mathrm{e}^{(bx+a)^2}}{4\,b^2} - \frac{a\mathrm{e}^{(bx+a)^2}}{4\,b^3} - \frac{\frac{i}{4}a^2\sqrt{\pi}\text{Erf}\,(ibx+ia)}{b^3} + \frac{\frac{i}{8}\sqrt{\pi}\text{Er}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh((b*x+a)^2),x)
```

```
[Out] 1/4/b^2*x*exp(-(b*x+a)^2)-1/4*a/b^3*exp(-(b*x+a)^2)-1/4*a^2*erf(b*x+a)*Pi^(
1/2)/b^3-1/8*erf(b*x+a)*Pi^(1/2)/b^3+1/4/b^2*x*exp((b*x+a)^2)-1/4*a/b^3*exp
((b*x+a)^2)-1/4*I*a^2/b^3*Pi^(1/2)*erf(I*b*x+I*a)+1/8*I/b^3*Pi^(1/2)*erf(I*
```

```
b*x+I*a)
```

Maxima [B] time = 1.81747, size = 1192, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh((b*x + a)^2) + 1/6*((sqrt(pi)*(b^2*x + a*b)*a^3*b^3*(erf(sqrt(
-(b^2*x + a*b)^2/b^2)) - 1)/((b^2)^(7/2)*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^
2*b^4*e^((b^2*x + a*b)^2/b^2)/(b^2)^(7/2) + b^4*gamma(2, -(b^2*x + a*b)^2/b
^2)/(b^2)^(7/2) - 3*(b^2*x + a*b)^3*a*b*gamma(3/2, -(b^2*x + a*b)^2/b^2)/((
b^2)^(7/2)*(-(b^2*x + a*b)^2/b^2)^(3/2)))*a/sqrt(b^2) + (sqrt(pi)*(b^2*x +
a*b)*a^3*b^3*(erf(sqrt((b^2*x + a*b)^2/b^2)) - 1)/((-b^2)^(7/2)*sqrt((b^2*x
 + a*b)^2/b^2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gam
ma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) - 3*(b^2*x + a*b)^3*a*b*gamma(3/2,
(b^2*x + a*b)^2/b^2)/((-b^2)^(7/2)*((b^2*x + a*b)^2/b^2)^(3/2)))*a/sqrt(-b^
2) + (sqrt(pi)*(b^2*x + a*b)*a^4*b^4*(erf(sqrt((b^2*x + a*b)^2/b^2)) - 1)/(
(-b^2)^(9/2)*sqrt((b^2*x + a*b)^2/b^2)) + 4*a^3*b^5*e^(-(b^2*x + a*b)^2/b^2
)/(-b^2)^(9/2) + 4*a*b^5*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(9/2) - 6*(b^
2*x + a*b)^3*a^2*b^2*gamma(3/2, (b^2*x + a*b)^2/b^2)/((-b^2)^(9/2)*((b^2*x
+ a*b)^2/b^2)^(3/2)) - (b^2*x + a*b)^5*gamma(5/2, (b^2*x + a*b)^2/b^2)/((-b
^2)^(9/2)*((b^2*x + a*b)^2/b^2)^(5/2)))*b/sqrt(-b^2) - (sqrt(pi)*(b^2*x + a
*b)*a^4*b^4*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/((b^2)^(9/2)*sqrt(-(b^2*x
 + a*b)^2/b^2)) - 4*a^3*b^5*e^((b^2*x + a*b)^2/b^2)/(b^2)^(9/2) + 4*a*b^5*g
amma(2, -(b^2*x + a*b)^2/b^2)/(b^2)^(9/2) - 6*(b^2*x + a*b)^3*a^2*b^2*gamma
(3/2, -(b^2*x + a*b)^2/b^2)/((b^2)^(9/2)*(-(b^2*x + a*b)^2/b^2)^(3/2)) - (b
^2*x + a*b)^5*gamma(5/2, -(b^2*x + a*b)^2/b^2)/((b^2)^(9/2)*(-(b^2*x + a*b)
^2/b^2)^(5/2)))*b/sqrt(b^2))*b
```

Fricas [A] time = 1.89151, size = 383, normalized size = 3.39

$$-\frac{\left(\sqrt{\pi}\left(2\,a^2+1\right)\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-\sqrt{\pi}\left(2\,a^2-1\right)\sqrt{b^2}\,\mathrm{erfi}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-2\,b^2x+2\,ab-2\,(b}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(2*a^2 + 1)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2
+ 2*a*b*x + a^2) - sqrt(pi)*(2*a^2 - 1)*sqrt(b^2)*erfi(sqrt(b^2)*(b*x + a)/
b)*e^(b^2*x^2 + 2*a*b*x + a^2) - 2*b^2*x + 2*a*b - 2*(b^2*x - a*b)*e^(2*b^2
*x^2 + 4*a*b*x + 2*a^2))*e^(-b^2*x^2 - 2*a*b*x - a^2)/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sinh\left(a^2 + 2abx + b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh((b*x+a)**2),x)
```

```
[Out] Integral(x**2*sinh(a**2 + 2*a*b*x + b**2*x**2), x)
```

Giac [C] time = 1.28316, size = 185, normalized size = 1.64

$$-\frac{\frac{i\sqrt{\pi}\left(2a^2-1\right)\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b} - \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(b^2x^2+2abx+a^2\right)}}{b}}{8b^2} + \frac{\frac{\sqrt{\pi}\left(2a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(-b^2x^2-2abx-a^2\right)}}{b}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(pi)*(2*a^2 - 1)*erf(I*b*(x + a/b))/b - 2*(b*(x + a/b) - 2*a)*e
^(b^2*x^2 + 2*a*b*x + a^2)/b)/b^2 + 1/8*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a
/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b^2
```

3.89 $\int x \sinh\left((a+bx)^2\right) dx$

Optimal. Leaf size=54

$$\frac{\sqrt{\pi}a\text{Erf}(a+bx)}{4b^2}-\frac{\sqrt{\pi}a\text{Erfi}(a+bx)}{4b^2}+\frac{\cosh\left((a+bx)^2\right)}{2b^2}$$

```
[Out] Cosh[(a + b*x)^2]/(2*b^2) + (a*Sqrt[Pi]*Erf[a + b*x])/(4*b^2) - (a*Sqrt[Pi]
*Erfi[a + b*x])/(4*b^2)
```

Rubi [A] time = 0.0532086, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5364, 6742, 5298, 2204, 2205, 5320, 2638}

$$\frac{\sqrt{\pi}a\text{Erf}(a+bx)}{4b^2}-\frac{\sqrt{\pi}a\text{Erfi}(a+bx)}{4b^2}+\frac{\cosh\left((a+bx)^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[(a + b*x)^2],x]
```

```
[Out] Cosh[(a + b*x)^2]/(2*b^2) + (a*Sqrt[Pi]*Erf[a + b*x])/(4*b^2) - (a*Sqrt[Pi]
*Erfi[a + b*x])/(4*b^2)
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh\left((a+bx)^2\right)dx &= \frac{\operatorname{Subst}\left(\int(-a+x)\sinh\left(x^2\right)dx,x,a+bx\right)}{b^2}\\
&= \frac{\operatorname{Subst}\left(\int\left(-a\sinh\left(x^2\right)+x\sinh\left(x^2\right)\right)dx,x,a+bx\right)}{b^2}\\
&= \frac{\operatorname{Subst}\left(\int x\sinh\left(x^2\right)dx,x,a+bx\right)}{b^2}-\frac{a\operatorname{Subst}\left(\int\sinh\left(x^2\right)dx,x,a+bx\right)}{b^2}\\
&= \frac{\operatorname{Subst}\left(\int\sinh(x)\,dx,x,(a+bx)^2\right)}{2b^2}+\frac{a\operatorname{Subst}\left(\int e^{-x^2}dx,x,a+bx\right)}{2b^2}-\frac{a\operatorname{Subst}\left(\int e^{x^2}dx,x,a+\right.}{2b^2}\\
&= \frac{\cosh\left((a+bx)^2\right)}{2b^2}+\frac{a\sqrt{\pi}\operatorname{erf}(a+bx)}{4b^2}-\frac{a\sqrt{\pi}\operatorname{erfi}(a+bx)}{4b^2}
\end{aligned}
$$

Mathematica [A] time = 0.0269452, size = 44, normalized size = 0.81

$$
\frac{\cosh\left((a+bx)^2\right)}{2b^2}-\frac{\sqrt{\pi}a(\operatorname{Erfi}(a+bx)-\operatorname{Erf}(a+bx))}{4b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[(a + b*x)^2],x]
```

```
[Out] Cosh[(a + b*x)^2]/(2*b^2) - (a*Sqrt[Pi]*(-Erf[a + b*x] + Erfi[a + b*x]))/(4
*b^2)
```

Maple [C] time = 0.029, size = 66, normalized size = 1.2

$$\frac{e^{-(bx+a)^2}}{4b^2}+\frac{a\mathrm{Erf}(bx+a)\sqrt{\pi}}{4b^2}+\frac{e^{(bx+a)^2}}{4b^2}+\frac{\frac{i}{4}a\sqrt{\pi}\mathrm{Erf}(ibx+ia)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh((b*x+a)^2),x)
```

```
[Out] 1/4/b^2*exp(-(b*x+a)^2)+1/4*a*erf(b*x+a)*Pi^(1/2)/b^2+1/4/b^2*exp((b*x+a)^2
)+1/4*I*a/b^2*Pi^(1/2)*erf(I*b*x+I*a)
```

Maxima [B] time = 1.67452, size = 948, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh((b*x + a)^2) - 1/4*((sqrt(pi)*(b^2*x + a*b)*a^2*b^2*(erf(sqrt(
-(b^2*x + a*b)^2/b^2)) - 1)/((b^2)^(5/2)*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*
b^3*e^((b^2*x + a*b)^2/b^2)/(b^2)^(5/2) - (b^2*x + a*b)^3*gamma(3/2, -(b^2*
x + a*b)^2/b^2)/((b^2)^(5/2)*(-(b^2*x + a*b)^2/b^2)^(3/2)))*a/sqrt(b^2) - (
sqrt(pi)*(b^2*x + a*b)*a^2*b^2*(erf(sqrt((b^2*x + a*b)^2/b^2)) - 1)/((-b^2)
^(5/2)*sqrt((b^2*x + a*b)^2/b^2)) + 2*a*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)
^(5/2) - (b^2*x + a*b)^3*gamma(3/2, (b^2*x + a*b)^2/b^2)/((-b^2)^(5/2)*((b^
2*x + a*b)^2/b^2)^(3/2)))*a/sqrt(-b^2) - (sqrt(pi)*(b^2*x + a*b)*a^3*b^3*(e
rf(sqrt((b^2*x + a*b)^2/b^2)) - 1)/((-b^2)^(7/2)*sqrt((b^2*x + a*b)^2/b^2))
 + 3*a^2*b^4*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (b^2*x +
a*b)^2/b^2)/(-b^2)^(7/2) - 3*(b^2*x + a*b)^3*a*b*gamma(3/2, (b^2*x + a*b)^2
/b^2)/((-b^2)^(7/2)*((b^2*x + a*b)^2/b^2)^(3/2)))*b/sqrt(-b^2) - (sqrt(pi)*
```

```
(b^2*x + a*b)*a^3*b^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/((b^2)^(7/2)*sq
rt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*b^4*e^((b^2*x + a*b)^2/b^2)/(b^2)^(7/2) +
 b^4*gamma(2, -(b^2*x + a*b)^2/b^2)/(b^2)^(7/2) - 3*(b^2*x + a*b)^3*a*b*gam
ma(3/2, -(b^2*x + a*b)^2/b^2)/((b^2)^(7/2)*(-(b^2*x + a*b)^2/b^2)^(3/2)))*b
/sqrt(b^2))*b
```

Fricas [B] time = 1.8869, size = 317, normalized size = 5.87

$$\frac{\left(\sqrt{\pi}a\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)} - \sqrt{\pi}a\sqrt{b^2}\,\mathrm{erfi}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)} + be^{\left(2\,b^2x^2+4\,abx+2\,a^2\right)} + b\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*a*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 + 2*a*b*x +
 a^2) - sqrt(pi)*a*sqrt(b^2)*erfi(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 + 2*a*b
*x + a^2) + b*e^(2*b^2*x^2 + 4*a*b*x + 2*a^2) + b)*e^(-b^2*x^2 - 2*a*b*x -
a^2)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh\left(a^2 + 2abx + b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)**2),x)
```

```
[Out] Integral(x*sinh(a**2 + 2*a*b*x + b**2*x**2), x)
```

Giac [C] time = 1.15458, size = 134, normalized size = 2.48

$$-\frac{-\frac{i\sqrt{\pi}a\,\mathrm{erf}\left(i\,b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(b^2x^2+2\,abx+a^2\right)}}{b}}{4\,b} - \frac{\frac{\sqrt{\pi}a\,\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(pi)*a*erf(I*b*(x + a/b))/b - e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b
 - 1/4*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b
```

3.90 $\int \sinh\left((a+bx)^2\right) dx$

Optimal. Leaf size=37

$$\frac{\sqrt{\pi}\text{Erfi}(a+bx)}{4b}-\frac{\sqrt{\pi}\text{Erf}(a+bx)}{4b}$$

```
[Out] -(Sqrt[Pi]*Erf[a + b*x])/(4*b) + (Sqrt[Pi]*Erfi[a + b*x])/(4*b)
```

Rubi [A] time = 0.0170969, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5310, 5298, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erfi}(a+bx)}{4b}-\frac{\sqrt{\pi}\text{Erf}(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)^2],x]
```

```
[Out] -(Sqrt[Pi]*Erf[a + b*x])/(4*b) + (Sqrt[Pi]*Erfi[a + b*x])/(4*b)
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sinh\left((a+bx)^2\right)dx &= \frac{\text{Subst}\left(\int \sinh\left(x^2\right)dx, x, a+bx\right)}{b}\\ &= -\frac{\text{Subst}\left(\int e^{-x^2}dx, x, a+bx\right)}{2b}+\frac{\text{Subst}\left(\int e^{x^2}dx, x, a+bx\right)}{2b}\\ &= -\frac{\sqrt{\pi}\,\text{erf}(a+bx)}{4b}+\frac{\sqrt{\pi}\,\text{erfi}(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0045263, size = 27, normalized size = 0.73

$$\frac{\sqrt{\pi}(\text{Erfi}(a+bx)-\text{Erf}(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi]*(-Erf[a + b*x] + Erfi[a + b*x]))/(4*b)
```

Maple [C] time = 0.028, size = 36, normalized size = 1.

$$-\frac{\text{Erf}\,(bx+a)\,\sqrt{\pi}}{4\,b}-\frac{\frac{i}{4}\sqrt{\pi}\,\text{Erf}\,(ibx+ia)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)^2),x)
```

```
[Out] -1/4*erf(b*x+a)*Pi^(1/2)/b-1/4*I*Pi^(1/2)/b*erf(I*b*x+I*a)
```

Maxima [B] time = 1.61128, size = 695, normalized size = 18.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((sqrt(pi)*(b^2*x + a*b)*a*b*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/((b^
2)^(3/2)*sqrt(-(b^2*x + a*b)^2/b^2)) - b^2*e^((b^2*x + a*b)^2/b^2)/(b^2)^(3
/2))*a/sqrt(b^2) + (sqrt(pi)*(b^2*x + a*b)*a*b*(erf(sqrt((b^2*x + a*b)^2/b^
2)) - 1)/((-b^2)^(3/2)*sqrt((b^2*x + a*b)^2/b^2)) + b^2*e^(-(b^2*x + a*b)^2
/b^2)/(-b^2)^(3/2))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^2*b^2*(erf(sqr
t((b^2*x + a*b)^2/b^2)) - 1)/((-b^2)^(5/2)*sqrt((b^2*x + a*b)^2/b^2)) + 2*a
*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2) - (b^2*x + a*b)^3*gamma(3/2, (b^
2*x + a*b)^2/b^2)/((-b^2)^(5/2)*((b^2*x + a*b)^2/b^2)^(3/2)))*b/sqrt(-b^2)
- (sqrt(pi)*(b^2*x + a*b)*a^2*b^2*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/((b
^2)^(5/2)*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*b^3*e^((b^2*x + a*b)^2/b^2)/(b^
2)^(5/2) - (b^2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/((b^2)^(5/2)*(-
(b^2*x + a*b)^2/b^2)^(3/2)))*b/sqrt(b^2))*b + x*sinh((b*x + a)^2)
```

Fricas [A] time = 1.80087, size = 144, normalized size = 3.89

$$-\frac{\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-\sqrt{\pi}\sqrt{b^2}\,\mathrm{erfi}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b) - sqrt(pi)*sqrt(b^2)*er
fi(sqrt(b^2)*(b*x + a)/b))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left((a+bx)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)**2),x)
```

```
[Out] Integral(sinh((a + b*x)**2), x)
```

Giac [C] time = 1.25509, size = 53, normalized size = 1.43

$$-\frac{i\sqrt{\pi}\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{4b}+\frac{\sqrt{\pi}\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(pi)*erf(I*b*(x + a/b))/b + 1/4*sqrt(pi)*erf(-b*(x + a/b))/b
```

3.91 $\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$

Optimal. Leaf size=19

$$b\text{CannotIntegrate}\left(\frac{\sinh\left((a+bx)^2\right)}{bx},x\right)$$

```
[Out] b*CannotIntegrate[Sinh[(a + b*x)^2]/(b*x), x]
```

Rubi [A] time = 0.035919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[(a + b*x)^2]/x,x]
```

```
[Out] Defer[Subst][Defer[Int][Sinh[x^2]/(-a + x), x], x, a + b*x]
```

Rubi steps

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx = \text{Subst}\left(\int \frac{\sinh\left(x^2\right)}{-a+x}\,dx,x,a+bx\right)$$

Mathematica [A] time = 9.85642, size = 0, normalized size = 0.

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[(a + b*x)^2]/x,x]
```

```
[Out] Integrate[Sinh[(a + b*x)^2]/x, x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\sinh\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)^2)/x,x)
```

```
[Out] int(sinh((b*x+a)^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(b^2x^2+2\,abx+a^2\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a^2 + 2abx + b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)**2)/x,x)
```

```
[Out] Integral(sinh(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((bx + a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh((b*x + a)^2)/x, x)
```

3.92 $\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sinh\left((a+bx)^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable[Sinh[(a + b*x)^2]/x^2, x]
```

Rubi [A] time = 0.0385763, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[(a + b*x)^2]/x^2,x]
```

```
[Out] b*Defer[Subst][Defer[Int][Sinh[x^2]/(-a + x)^2, x], x, a + b*x]
```

Rubi steps

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx = b\,\text{Subst}\left(\int \frac{\sinh\left(x^2\right)}{(-a+x)^2} dx, x, a+bx\right)$$

Mathematica [A] time = 13.156, size = 0, normalized size = 0.

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[(a + b*x)^2]/x^2,x]
```

```
[Out] Integrate[Sinh[(a + b*x)^2]/x^2, x]
```

Maple [A] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)^2)/x^2,x)
```

```
[Out] int(sinh((b*x+a)^2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)^2)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sinh\left(b^2x^2+2\,abx+a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a^2+2abx+b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)**2)/x**2,x)
```

```
[Out] Integral(sinh(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh((b*x + a)^2)/x^2, x)
```

3.93 $\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=346

$$-\frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\sinh\left(a+\right.}{b^4d^3}$$

```
[Out] (240*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^5*d^3) - (24*c*Sqrt[c + d*
x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Cosh[a + b*S
qrt[c + d*x]])/(b*d^3) + (40*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b^
3*d^3) - (4*c*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c +
d*x)^(5/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) - (240*Sinh[a + b*Sqrt[c + d*
x]])/(b^6*d^3) + (24*c*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (2*c^2*Sinh[a
 + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/
(b^4*d^3) + (12*c*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (10*(c +
 d*x)^2*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```

Rubi [A] time = 0.418116, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5364, 5286, 3296, 2637}

$$-\frac{2c^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{10(c+dx)^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{12c(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\sinh\left(a+\right.}{b^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*Sqrt[c + d*x]],x]
```


```
[Out] (240*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^5*d^3) - (24*c*Sqrt[c + d*
x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Cosh[a + b*S
qrt[c + d*x]])/(b*d^3) + (40*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b^
3*d^3) - (4*c*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c +
d*x)^(5/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) - (240*Sinh[a + b*Sqrt[c + d*
x]])/(b^6*d^3) + (24*c*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (2*c^2*Sinh[a
 + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/
(b^4*d^3) + (12*c*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (10*(c +
 d*x)^2*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```


Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
```

```
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2 \sinh\left(a+b\sqrt{c+dx}\right) dx = \frac{\text{Subst}\left(\int (-c+x)^2 \sinh\left(a+b\sqrt{x}\right) dx, x, c+dx\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int x\left(c-x^2\right)^2 \sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int \left(c^2x\sinh(a+bx) - 2cx^3\sinh(a+bx) + x^5\sinh(a+bx)\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int x^5\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} - \frac{(4c)\,\text{Subst}\left(\int x^3\sinh(a+bx)\,dx, x, \sqrt{c+a}\right.}{d^3}$$
$$= \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cos}{b}$$
$$= \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\cos}{b}$$
$$= -\frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c+dx)^{3/2}\,c}{}$$
$$= -\frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c+dx)^{3/2}\,c}{}$$
$$= \frac{240\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} - \frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh}{b}$$
$$= \frac{240\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} - \frac{24c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\cosh}{b}$$

Mathematica [A] time = 1.29631, size = 104, normalized size = 0.3

$$\frac{2b\sqrt{c+dx}\left(4b^2(2c+5dx)+b^4d^2x^2+120\right)\cosh\left(a+b\sqrt{c+dx}\right) - 2\left(b^4dx(4c+5dx)+12b^2(4c+5dx)+120\right)\sinh\left(a+b\right.}{b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[c + d*x]*(120 + b^4*d^2*x^2 + 4*b^2*(2*c + 5*d*x))*Cosh[a + b*Sqr
t[c + d*x]] - 2*(120 + 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x))*Sinh[a
 + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Maple [B] time = 0.01, size = 831, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^3/b^2*(1/b^4*((a+b*(d*x+c)^(1/2))^5*cosh(a+b*(d*x+c)^(1/2))-5*(a+b*(d*x
+c)^(1/2))^4*sinh(a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x
+c)^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))+120*(a+b*(d*x+c
)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-120*sinh(a+b*(d*x+c)^(1/2)))-5/b^4*a*((a+b
*(d*x+c)^(1/2))^4*cosh(a+b*(d*x+c)^(1/2))-4*(a+b*(d*x+c)^(1/2))^3*sinh(a+b*
(d*x+c)^(1/2))+12*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-24*(a+b*(d*
x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+24*cosh(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*
((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*sinh
(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-6*sinh(a+
b*(d*x+c)^(1/2)))-10/b^4*a^3*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))
-2*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/2)))-2
/b^2*c*((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))
^2*sinh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-6*
sinh(a+b*(d*x+c)^(1/2)))+6/b^2*c*a*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^
(1/2))-2*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/
2)))+5/b^4*a^4*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c
)^(1/2)))-6/b^2*a^2*c*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b
*(d*x+c)^(1/2)))-1/b^4*a^5*cosh(a+b*(d*x+c)^(1/2))+2/b^2*a^3*c*cosh(a+b*(d*
x+c)^(1/2))+c^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+
c)^(1/2)))-c^2*a*cosh(a+b*(d*x+c)^(1/2)))
```

Maxima [A] time = 1.09458, size = 656, normalized size = 1.9

$$2\,d^3x^3\sinh\left(\sqrt{dx+c}\,b+a\right)+\left(\frac{c^3e^{\left(\sqrt{dx+c}b+a\right)}}{b}-\frac{c^3e^{\left(-\sqrt{dx+c}b-a\right)}}{b}-\frac{3\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2\,e^a\right)c^2e^{\left(\sqrt{dx+c}b\right)}}{b^3}+\frac{3\left((dx+c)b^2+2\sqrt{dx+c}b+2\right)c^2e^{(-}}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*sinh(sqrt(d*x + c)*b + a) + (c^3*e^(sqrt(d*x + c)*b + a)/b -
 c^3*e^(-sqrt(d*x + c)*b - a)/b - 3*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c^2*e^(sqrt(d*x + c)*b)/b^3 + 3*((d*x + c)*b^2 + 2*sqrt(d*x +
c)*b + 2)*c^2*e^(-sqrt(d*x + c)*b - a)/b^3 + 3*((d*x + c)^2*b^4*e^a - 4*(d*
x + c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e
^a)*c*e^(sqrt(d*x + c)*b)/b^5 - 3*((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3
```

```
+ 12*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*c*e^(-sqrt(d*x + c)*b - a)/b^
5 - ((d*x + c)^3*b^6*e^a - 6*(d*x + c)^(5/2)*b^5*e^a + 30*(d*x + c)^2*b^4*e
^a - 120*(d*x + c)^(3/2)*b^3*e^a + 360*(d*x + c)*b^2*e^a - 720*sqrt(d*x + c
)*b*e^a + 720*e^a)*e^(sqrt(d*x + c)*b)/b^7 + ((d*x + c)^3*b^6 + 6*(d*x + c)
^(5/2)*b^5 + 30*(d*x + c)^2*b^4 + 120*(d*x + c)^(3/2)*b^3 + 360*(d*x + c)*b
^2 + 720*sqrt(d*x + c)*b + 720)*e^(-sqrt(d*x + c)*b - a)/b^7)*b)/d^3
```

Fricas [A] time = 2.09057, size = 251, normalized size = 0.73

$$\frac{2\left(\left(b^5d^2x^2 + 20\,b^3dx + 8\,b^3c + 120\,b\right)\sqrt{dx+c}\cosh\left(\sqrt{dx+c}b + a\right) - \left(5\,b^4d^2x^2 + 48\,b^2c + 4\left(b^4c + 15\,b^2\right)dx + 120\right)\sinh\left(\right.}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^5*d^2*x^2 + 20*b^3*d*x + 8*b^3*c + 120*b)*sqrt(d*x + c)*cosh(sqrt(d*x
 + c)*b + a) - (5*b^4*d^2*x^2 + 48*b^2*c + 4*(b^4*c + 15*b^2)*d*x + 120)*si
nh(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

Sympy [A] time = 2.4445, size = 269, normalized size = 0.78

$$\begin{cases} \frac{x^3\sinh(a)}{3} \\ \frac{x^3\sinh\left(a+b\sqrt{c}\right)}{3} \\ \frac{2x^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{8cx\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{10x^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{16c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{40x\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{96c\,\mathrm{si}}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**3*sinh(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*sinh(a
+ b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/
(b*d) - 8*c*x*sinh(a + b*sqrt(c + d*x))/(b**2*d**2) - 10*x**2*sinh(a + b*sq
rt(c + d*x))/(b**2*d) + 16*c*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*
d**3) + 40*x*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*sin
h(a + b*sqrt(c + d*x))/(b**4*d**3) - 120*x*sinh(a + b*sqrt(c + d*x))/(b**4*
d**2) + 240*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**5*d**3) - 240*sinh(
```

```
a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

Giac [B] time = 1.90568, size = 2072, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - b^4*c^2*sgn((sqrt(d*x + c)*b
+ a)*b - a*b) - 2*(sqrt(d*x + c)*b + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2
*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c + 2*a^3*b^2*c + 6*(sqrt(d*x +
c)*b + a)^2*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 12*(sqrt(d*x + c)*b
+ a)*a*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 6*a^2*b^2*c*sgn((sqrt(d*x
 + c)*b + a)*b - a*b) + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)^4
*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*(s
qrt(d*x + c)*b + a)*a^4 - a^5 - 5*(sqrt(d*x + c)*b + a)^4*sgn((sqrt(d*x + c
)*b + a)*b - a*b) + 20*(sqrt(d*x + c)*b + a)^3*a*sgn((sqrt(d*x + c)*b + a)*
b - a*b) - 30*(sqrt(d*x + c)*b + a)^2*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b
) + 20*(sqrt(d*x + c)*b + a)*a^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 5*a^4
*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a
*b^2*c + 12*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 20*(sqrt(d*x + c)*b
+ a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*x + c)*b + a)*a^2 - 20*a
^3 - 60*(sqrt(d*x + c)*b + a)^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 120*(s
qrt(d*x + c)*b + a)*a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 60*a^2*sgn((sqrt
(d*x + c)*b + a)*b - a*b) + 120*sqrt(d*x + c)*b - 120*sgn((sqrt(d*x + c)*b
+ a)*b - a*b))*e^((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b)
- a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)/(b^5*d^2) + ((sqrt(d*x + c)*b +
 a)*b^4*c^2 - a*b^4*c^2 + b^4*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 2*(s
qrt(d*x + c)*b + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d
*x + c)*b + a)*a^2*b^2*c + 2*a^3*b^2*c - 6*(sqrt(d*x + c)*b + a)^2*b^2*c*sg
n((sqrt(d*x + c)*b + a)*b - a*b) + 12*(sqrt(d*x + c)*b + a)*a*b^2*c*sgn((sq
rt(d*x + c)*b + a)*b - a*b) - 6*a^2*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b
) + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)^4*a + 10*(sqrt(d*x +
c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*(sqrt(d*x + c)*b + a)*
a^4 - a^5 + 5*(sqrt(d*x + c)*b + a)^4*sgn((sqrt(d*x + c)*b + a)*b - a*b) -
20*(sqrt(d*x + c)*b + a)^3*a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 30*(sqrt(
d*x + c)*b + a)^2*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 20*(sqrt(d*x + c
)*b + a)*a^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 5*a^4*sgn((sqrt(d*x + c)*
b + a)*b - a*b) - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*c - 12*b^2*c*sg
n((sqrt(d*x + c)*b + a)*b - a*b) + 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*
```

```
x + c)*b + a)^2*a + 60*(sqrt(d*x + c)*b + a)*a^2 - 20*a^3 + 60*(sqrt(d*x +
c)*b + a)^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 120*(sqrt(d*x + c)*b + a)*
a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 60*a^2*sgn((sqrt(d*x + c)*b + a)*b -
 a*b) + 120*sqrt(d*x + c)*b + 120*sgn((sqrt(d*x + c)*b + a)*b - a*b))*e^(-(
sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x +
 c)*b + a)*b - a*b) - a)/(b^5*d^2))/(b*d)
```

3.94 $\int x \sinh \left(a + b\sqrt{c + dx}\right) dx$

Optimal. Leaf size=167

$$-\frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}$$

```
[Out] (12*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]
*Cosh[a + b*Sqrt[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c
+ d*x]])/(b*d^2) - (12*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (2*c*Sinh[a +
 b*Sqrt[c + d*x]])/(b^2*d^2) - (6*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2
*d^2)
```

Rubi [A] time = 0.186434, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5364, 5286, 3296, 2637}

$$-\frac{6(c+dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (12*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]
*Cosh[a + b*Sqrt[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c
+ d*x]])/(b*d^2) - (12*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (2*c*Sinh[a +
 b*Sqrt[c + d*x]])/(b^2*d^2) - (6*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2
*d^2)
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x\sinh\left(a+b\sqrt{c+dx}\right)dx &= \frac{\text{Subst}\left(\int(-c+x)\sinh\left(a+b\sqrt{x}\right)dx,x,c+dx\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int x\left(-c+x^2\right)\sinh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int\left(-cx\sinh(a+bx)+x^3\sinh(a+bx)\right)dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int x^3\sinh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}-\frac{(2c)\,\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,\sqrt{c+dx}\right.}{d^2}\\
&= -\frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{6\,\text{Subst}\left(\int x^2\,\text{co}\right.}{}\\
&= -\frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2c\sinh\left(a+b\sqrt{c\right.}}{b^2d^2}\\
&= \frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\cosh\left(a\right.}{bd^2}\\
&= \frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\cosh\left(a\right.}{bd^2}
\end{aligned}$$

Mathematica [A] time = 0.198569, size = 72, normalized size = 0.43

$$\frac{2b\left(b^2dx+6\right)\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)-2\left(b^2(2c+3dx)+6\right)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[c + d*x]*(6 + b^2*d*x)*Cosh[a + b*Sqrt[c + d*x]] - 2*(6 + b^2*(2*
c + 3*d*x))*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [B] time = 0.01, size = 303, normalized size = 1.8

$$2\,\frac{1}{d^2b^2}\left(\frac{\left(a+b\sqrt{dx+c}\right)^3\cosh\left(a+b\sqrt{dx+c}\right)-3\,\left(a+b\sqrt{dx+c}\right)^2\sinh\left(a+b\sqrt{dx+c}\right)+6\,\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^2/b^2*(1/b^2*((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x
+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)
^(1/2))-6*sinh(a+b*(d*x+c)^(1/2)))-3/b^2*a*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*
(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*
x+c)^(1/2)))+3/b^2*a^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+
b*(d*x+c)^(1/2)))-1/b^2*a^3*cosh(a+b*(d*x+c)^(1/2))-c*((a+b*(d*x+c)^(1/2))*
cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)))+c*a*cosh(a+b*(d*x+c)^(1/2)
))
```

Maxima [A] time = 1.22019, size = 396, normalized size = 2.37

$$\frac{2\,d^2x^2\sinh\left(\sqrt{dx+c}b+a\right)-\left(\frac{c^2e^{\left(\sqrt{dx+c}b+a\right)}}{b}-\frac{c^2e^{\left(-\sqrt{dx+c}b-a\right)}}{b}-\frac{2\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2e^a\right)ce^{\left(\sqrt{dx+c}b\right)}}{b^3}+\frac{2\left((dx+c)b^2+2\sqrt{dx+c}b+2\right)ce^{\left(-\sqrt{d\right.}}}{b^3}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*sinh(sqrt(d*x + c)*b + a) - (c^2*e^(sqrt(d*x + c)*b + a)/b -
 c^2*e^(-sqrt(d*x + c)*b - a)/b - 2*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c*e^(sqrt(d*x + c)*b)/b^3 + 2*((d*x + c)*b^2 + 2*sqrt(d*x + c)
*b + 2)*c*e^(-sqrt(d*x + c)*b - a)/b^3 + ((d*x + c)^2*b^4*e^a - 4*(d*x + c)
```

```
^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e^a)*e^
(sqrt(d*x + c)*b)/b^5 - ((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3 + 12*(d*x
+ c)*b^2 + 24*sqrt(d*x + c)*b + 24)*e^(-sqrt(d*x + c)*b - a)/b^5)*b)/d^2
```

Fricas [A] time = 2.03561, size = 169, normalized size = 1.01

$$\frac{2\left(\left(b^3dx+6b\right)\sqrt{dx+c}\cosh\left(\sqrt{dx+c}b+a\right)-\left(3b^2dx+2b^2c+6\right)\sinh\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^3*d*x + 6*b)*sqrt(d*x + c)*cosh(sqrt(d*x + c)*b + a) - (3*b^2*d*x + 2
*b^2*c + 6)*sinh(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

Sympy [A] time = 0.770839, size = 151, normalized size = 0.9

$$\begin{cases} \frac{x^2\sinh(a)}{2} & \text{for } b=0 \wedge (b \\ \frac{x^2\sinh\left(a+b\sqrt{c}\right)}{2} & \text{for } d=0 \\ \frac{2x\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{4c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6x\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{12\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**2*sinh(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*sinh(a
+ b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b*
d) - 4*c*sinh(a + b*sqrt(c + d*x))/(b**2*d**2) - 6*x*sinh(a + b*sqrt(c + d*
x))/(b**2*d) + 12*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*
sinh(a + b*sqrt(c + d*x))/(b**4*d**2), True))
```

Giac [B] time = 1.57533, size = 780, normalized size = 4.67

$$-\frac{\frac{\left(\left(\sqrt{dx+c}b+a\right)b^2c-ab^2c-b^2c\,\mathrm{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-\left(\sqrt{dx+c}b+a\right)^3+3\left(\sqrt{dx+c}b+a\right)^2a-3\left(\sqrt{dx+c}b+a\right)a^2+a^3+3\left(\sqrt{dx+c}b+a\right)^2\mathrm{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-6\left(\sqrt{dx+cb}\right.}{b^3d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - b^2*c*sgn((sqrt(d*x + c)*b + a)*
b - a*b) - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(
d*x + c)*b + a)*a^2 + a^3 + 3*(sqrt(d*x + c)*b + a)^2*sgn((sqrt(d*x + c)*b
+ a)*b - a*b) - 6*(sqrt(d*x + c)*b + a)*a*sgn((sqrt(d*x + c)*b + a)*b - a*b
) + 3*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 6*sqrt(d*x + c)*b + 6*sgn((s
qrt(d*x + c)*b + a)*b - a*b))*e^((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b
 + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)/(b^3*d) + ((sqrt
(d*x + c)*b + a)*b^2*c - a*b^2*c + b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)
*b + a)*a^2 + a^3 - 3*(sqrt(d*x + c)*b + a)^2*sgn((sqrt(d*x + c)*b + a)*b -
 a*b) + 6*(sqrt(d*x + c)*b + a)*a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3*a^
2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 6*sqrt(d*x + c)*b - 6*sgn((sqrt(d*x
+ c)*b + a)*b - a*b))*e^(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b
 - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a)/(b^3*d))/(b*d)
```

3.95 $\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] (2*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b*d) - (2*Sinh[a + b*Sqrt[c +
d*x]])/(b^2*d)
```

Rubi [A] time = 0.0457324, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5310, 5304, 3296, 2637}

$$\frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b*d) - (2*Sinh[a + b*Sqrt[c +
d*x]])/(b^2*d)
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rule 5304

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Sinh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh\left(a+b\sqrt{c+dx}\right)dx &= \frac{\text{Subst}\left(\int \sinh\left(a+b\sqrt{x}\right)dx, x, c+dx\right)}{d} \\
&= \frac{2\,\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d} \\
&= \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\,\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd} \\
&= \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}
$$

Mathematica [A] time = 0.0626198, size = 50, normalized size = 0.93

$$\frac{2\left(b\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)-\sinh\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*(b*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]] - Sinh[a + b*Sqrt[c + d*x]]))
/(b^2*d)
```

Maple [A] time = 0.007, size = 63, normalized size = 1.2

$$2\,\frac{\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\sqrt{dx+c}\right)-\sinh\left(a+b\sqrt{dx+c}\right)-a\cosh\left(a+b\sqrt{dx+c}\right)}{db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d/b^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)
)-a*cosh(a+b*(d*x+c)^(1/2)))
```

Maxima [B] time = 1.14806, size = 150, normalized size = 2.78

$$-\frac{b\left(\frac{\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2e^a\right)e^{\left(\sqrt{dx+c}b\right)}}{b^3}-\frac{\left((dx+c)b^2+2\sqrt{dx+c}b+2\right)e^{\left(-\sqrt{dx+c}b-a\right)}}{b^3}\right)-2(dx+c)\sinh\left(\sqrt{dx+c}b+a\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*((((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*e^a + 2*e^a)*e^(sqrt(d*x +
c)*b)/b^3 - ((d*x + c)*b^2 + 2*sqrt(d*x + c)*b + 2)*e^(-sqrt(d*x + c)*b - a
)/b^3) - 2*(d*x + c)*sinh(sqrt(d*x + c)*b + a))/d
```

Fricas [A] time = 2.03717, size = 112, normalized size = 2.07

$$\frac{2\left(\sqrt{dx+c}b\cosh\left(\sqrt{dx+c}b+a\right)-\sinh\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*b*cosh(sqrt(d*x + c)*b + a) - sinh(sqrt(d*x + c)*b + a))/(
b^2*d)
```

Sympy [A] time = 0.628374, size = 65, normalized size = 1.2

$$\begin{cases} x\sinh(a) & \text{for } b=0 \wedge (b=0 \vee d=0) \\ x\sinh\left(a+b\sqrt{c}\right) & \text{for } d=0 \\ \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sinh(a + b*sqrt
(c)), Eq(d, 0)), (2*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b*d) - 2*sinh(
a + b*sqrt(c + d*x))/(b**2*d), True))
```

<u>Giac</u> [B] time = 1.32425, size = 281, normalized size = 5.2

$$\frac{\left(\left(\sqrt{dx+c}b+a\right)b-ab-b\operatorname{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)\right)e^{\left(\left(\sqrt{dx+c}b+a\right)\operatorname{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-a\operatorname{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)+a\right)}}{b^3d}+\frac{\left(\left(\sqrt{dx+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] ((sqrt(d*x + c)*b + a)*b - a*b - b*sgn((sqrt(d*x + c)*b + a)*b - a*b))*e^((
sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x +
 c)*b + a)*b - a*b) + a)/(b^3*d) + ((sqrt(d*x + c)*b + a)*b - a*b + b*sgn((
sqrt(d*x + c)*b + a)*b - a*b))*e^(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)
*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a)/(b^3*d)
```

3.96 $\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=124

$$\sinh\left(a-b\sqrt{c}\right)\mathrm{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\sinh\left(a+b\sqrt{c}\right)\mathrm{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)-\cosh\left(a+b\sqrt{c}\right)\mathrm{Shi}\left(b\left(\sqrt{c}-\sqrt{c+d\right.\right.$$

```
[Out] CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])]*Sinh[a - b*Sqrt[c]] + CoshIntegra
l[b*(Sqrt[c] - Sqrt[c + d*x])]*Sinh[a + b*Sqrt[c]] - Cosh[a + b*Sqrt[c]]*Si
nhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Cosh[a - b*Sqrt[c]]*SinhIntegral[
b*(Sqrt[c] + Sqrt[c + d*x])]
```

Rubi [A] time = 0.286707, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5364, 5292, 3303, 3298, 3301}

$$\sinh\left(a-b\sqrt{c}\right)\mathrm{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\sinh\left(a+b\sqrt{c}\right)\mathrm{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)-\cosh\left(a+b\sqrt{c}\right)\mathrm{Shi}\left(b\left(\sqrt{c}-\sqrt{c+d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])]*Sinh[a - b*Sqrt[c]] + CoshIntegra
l[b*(Sqrt[c] - Sqrt[c + d*x])]*Sinh[a + b*Sqrt[c]] - Cosh[a + b*Sqrt[c]]*Si
nhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Cosh[a - b*Sqrt[c]]*SinhIntegral[
b*(Sqrt[c] + Sqrt[c + d*x])]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt{x}\right)}{-c+x}\,dx, x, c+dx\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x\sinh(a+bx)}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-\frac{\sinh(a+bx)}{2\left(\sqrt{c}-x\right)}+\frac{\sinh(a+bx)}{2\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c+dx}\right)$$
$$= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)$$
$$= \cosh\left(a-b\sqrt{c}\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)+\cosh\left(a+b\sqrt{c}\right)\text{Subst}\left(\int \frac{s}{-}\right.$$
$$= \textbf{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)+\textbf{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)\sinh\left(a+b\sqrt{c}\right)+\cosh\left(\right.$$

Mathematica [A] time = 0.963794, size = 130, normalized size = 1.05

$$\frac{1}{2}e^{-a-b\sqrt{c}}\left(e^{2\left(a+b\sqrt{c}\right)}\text{ExpIntegralEi}\left(b\left(\sqrt{c+dx}-\sqrt{c}\right)\right)+e^{2a}\text{ExpIntegralEi}\left(b\left(\sqrt{c+dx}+\sqrt{c}\right)\right)-\text{ExpIntegralEi}\left(b\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] (E^(-a - b*Sqrt[c])*(-ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])] + E^(2*(a
+ b*Sqrt[c]))*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - E^(2*b*Sqrt[c])
*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + E^(2*a)*ExpIntegralEi[b*(S
qrt[c] + Sqrt[c + d*x])]))/2
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sinh\left(a+b\sqrt{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/2))/x,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/2))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x, x)
```

Fricas [B] time = 2.1562, size = 547, normalized size = 4.41

$$\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b+\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}b-\sqrt{b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b + sqrt(b^2*c))
)*cosh(a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - Ei(-sqrt
(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*
b - sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)
) - 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b - sqrt(b^2
*c)))*sinh(-a + sqrt(b^2*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + b\sqrt{c + dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2))/x,x)
```

```
[Out] Integral(sinh(a + b*sqrt(c + d*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(\sqrt{dx + c}b + a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x, x)
```

3.97 $\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=182

$$\frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}\right.\right.}{2\sqrt{c}}$$

```
[Out] (b*d*Cosh[a + b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt
[c]) - (b*d*Cosh[a - b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/
(2*Sqrt[c]) - Sinh[a + b*Sqrt[c + d*x]]/x - (b*d*Sinh[a + b*Sqrt[c]]*SinhIn
tegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt[c]) - (b*d*Sinh[a - b*Sqrt[c]]
*SinhIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c])
```

Rubi [A] time = 0.360614, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5364, 5288, 5281, 3303, 3298, 3301}

$$\frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}\right.\right.}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (b*d*Cosh[a + b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt
[c]) - (b*d*Cosh[a - b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/
(2*Sqrt[c]) - Sinh[a + b*Sqrt[c + d*x]]/x - (b*d*Sinh[a + b*Sqrt[c]]*SinhIn
tegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt[c]) - (b*d*Sinh[a - b*Sqrt[c]]
*SinhIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c])
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
```

```
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = d\,\text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)$$
$$= (2d)\,\text{Subst}\left(\int \frac{x\sinh(a+bx)}{\left(c-x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - (bd)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{c-x^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - (bd)\,\text{Subst}\left(\int \left(\frac{\cosh(a+bx)}{2\sqrt{c}\left(\sqrt{c}-x\right)} + \frac{\cosh(a+bx)}{2\sqrt{c}\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}+x}\right.}{2\sqrt{c}}$$
$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{(bd\,c}{}$$
$$= -\frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}} -$$

Mathematica [A] time = 3.04243, size = 199, normalized size = 1.09

$$\frac{e^{-a}\left(bdxe^{-b\sqrt{c}}\text{ExpIntegralEi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) - bdxe^{b\sqrt{c}}\text{ExpIntegralEi}\left(-b\left(\sqrt{c+dx}+\sqrt{c}\right)\right) + 2\sqrt{c}e^{-b\sqrt{c+dx}}\right) + e^{a}\left(b\right.}{4\sqrt{c}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (((2*Sqrt[c])/E^(b*Sqrt[c + d*x]) + (b*d*x*ExpIntegralEi[b*(Sqrt[c] - Sqrt[
c + d*x])])/E^(b*Sqrt[c]) - b*d*E^(b*Sqrt[c])*x*ExpIntegralEi[-(b*(Sqrt[c]
+ Sqrt[c + d*x]))])/E^a + E^a*(-2*Sqrt[c]*E^(b*Sqrt[c + d*x]) + b*d*E^(b*Sq
rt[c])*x*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - (b*d*x*ExpIntegralEi
[b*(Sqrt[c] + Sqrt[c + d*x])])/E^(b*Sqrt[c])))/(4*Sqrt[c]*x)
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sinh\left(a+b\sqrt{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/2))/x^2,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/2))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14669, size = 757, normalized size = 4.16

$$\left(\sqrt{b^2c}dx\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)+\sqrt{b^2c}dx\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)-\left(\sqrt{b^2c}dx\mathrm{Ei}\left(\sqrt{dx+c}b+\sqrt{b^2c}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + sqrt(b^2*c)*d*x*E
i(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x
*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b -
sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) - 4*c*sinh(sqrt(d*x + c)*b + a) + (sqr
t(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d
*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) + (sqrt(b^2*c)*d*x*Ei(sqrt(
d*x + c)*b + sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*
c)))*sinh(-a + sqrt(b^2*c)))/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2))/x**2,x)
```

```
[Out] Integral(sinh(a + b*sqrt(c + d*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(\sqrt{dx+c}\,b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x^2, x)
```

3.98 $\int x^2 \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=537

$$-\frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}+\frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{1008(c+dx)^{5/3}\,\mathrm{si}\cdots}{b^{\cdots}}$$

```
[Out] (120960*Cosh[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*c^2*Cosh[a + b*(c + d*x
)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)])/(
b^5*d^3) + (60480*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^7*d^3) +
(3*c^2*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) - (120*c*(c + d
*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (5040*(c + d*x)^(4/3)*Cosh[a +
 b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^
(1/3)])/(b*d^3) + (168*(c + d*x)^2*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) +
 (3*(c + d*x)^(8/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*c*Sinh[a +
b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x
)^(1/3)])/(b^8*d^3) - (6*c^2*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) + (360*c*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^3) -
(20160*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^3) + (30*c*(c + d*x)^(
4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*x)^(5/3)*Sinh[a
+ b*(c + d*x)^(1/3)])/(b^4*d^3) - (24*(c + d*x)^(7/3)*Sinh[a + b*(c + d*x)^
(1/3)])/(b^2*d^3)
```

Rubi [A] time = 0.696997, antiderivative size = 537, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5364, 1593, 5286, 3296, 2638, 2637}

$$-\frac{6c^2\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}+\frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{1008(c+dx)^{5/3}\,\mathrm{si}\cdots}{b^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (120960*Cosh[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*c^2*Cosh[a + b*(c + d*x
)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)])/(
b^5*d^3) + (60480*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^7*d^3) +
(3*c^2*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) - (120*c*(c + d
*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (5040*(c + d*x)^(4/3)*Cosh[a +
 b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^
(1/3)])/(b*d^3) + (168*(c + d*x)^2*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) +
```

```
(3*(c + d*x)^(8/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*c*Sinh[a +
b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x
)^(1/3)])/(b^8*d^3) - (6*c^2*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) + (360*c*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^3) -
(20160*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^3) + (30*c*(c + d*x)^(
4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*x)^(5/3)*Sinh[a
+ b*(c + d*x)^(1/3)])/(b^4*d^3) - (24*(c + d*x)^(7/3)*Sinh[a + b*(c + d*x)^
(1/3)])/(b^2*d^3)
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2 \sinh\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{\text{Subst}\left(\int (-c+x)^2 \sinh\left(a+b\sqrt[3]{x}\right) dx, x, c+dx\right)}{d^3}$$

$$= \frac{3\,\text{Subst}\left(\int \left(-cx+x^4\right)^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{3\,\text{Subst}\left(\int x^2\left(-c+x^3\right)^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{3\,\text{Subst}\left(\int \left(c^2x^2\sinh(a+bx) - 2cx^5\sinh(a+bx) + x^8\sinh(a+bx)\right) dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{3\,\text{Subst}\left(\int x^8 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} - \frac{(6c)\,\text{Subst}\left(\int x^5 \sinh(a+bx)\,dx, x, \sqrt[3]{c+\right.}{d^3}$$

$$= \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}}{}$$

$$= \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}}{}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a\right.}{b^3d^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\cosh\left(a\right.}{b^3d^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a\right.}{bd^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a\right.}{bd^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{60480(c+dx)^{2/3}\cosh}{b^7d^3}$$

$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{60480(c+dx)^{2/3}\cosh}{b^7d^3}$$

$$= \frac{120960\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} + \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{\right.}{b^5d^3}$$

Mathematica [A] time = 2.8738, size = 378, normalized size = 0.7

$$\frac{3\left((\sinh(a)+\cosh(a))\left(2b^6\left(9c^2+36cdx+28d^2x^2\right)+b^8d^2x^2(c+dx)^{2/3}-2b^7dx\sqrt[3]{c+dx}(3c+4dx)-24b^5(c+dx)^{2/3}(9c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*((40320 - 40320*b*(c + d*x)^(1/3) + 20160*b^2*(c + d*x)^(2/3) + b^8*d^2*
x^2*(c + d*x)^(2/3) - 2*b^7*d*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(c
+ d*x)^(1/3)*(6*c + 7*d*x) - 24*b^5*(c + d*x)^(2/3)*(9*c + 14*d*x) - 240*b^
3*(27*c + 28*d*x) + 2*b^6*(9*c^2 + 36*c*d*x + 28*d^2*x^2))*(Cosh[a] + Sinh[
a])*(Cosh[b*(c + d*x)^(1/3)] + Sinh[b*(c + d*x)^(1/3)]) + (40320 + 40320*b*
(c + d*x)^(1/3) + 20160*b^2*(c + d*x)^(2/3) + b^8*d^2*x^2*(c + d*x)^(2/3) +
 2*b^7*d*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(c + d*x)^(1/3)*(6*c + 7
*d*x) + 24*b^5*(c + d*x)^(2/3)*(9*c + 14*d*x) + 240*b^3*(27*c + 28*d*x) + 2
*b^6*(9*c^2 + 36*c*d*x + 28*d^2*x^2))*(Cosh[a + b*(c + d*x)^(1/3)] - Sinh[a
 + b*(c + d*x)^(1/3)])))/(2*b^9*d^3)
```

Maple [B] time = 0.01, size = 1815, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^3/b^3*(1/b^6*((a+b*(d*x+c)^(1/3))^8*cosh(a+b*(d*x+c)^(1/3))-8*(a+b*(d*x
+c)^(1/3))^7*sinh(a+b*(d*x+c)^(1/3))+56*(a+b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x
+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))+1680*(a+b*(d*x
+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-6720*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d
*x+c)^(1/3))+20160*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-40320*(a+b
*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+40320*cosh(a+b*(d*x+c)^(1/3)))+1/b^
6*a^8*cosh(a+b*(d*x+c)^(1/3))+c^2*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(
1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3
)))+c^2*a^2*cosh(a+b*(d*x+c)^(1/3))+2/b^3*a^5*c*cosh(a+b*(d*x+c)^(1/3))-56/
b^6*a^3*((a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3)
)^4*sinh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3)
)-60*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*
cosh(a+b*(d*x+c)^(1/3))-120*sinh(a+b*(d*x+c)^(1/3)))-2/b^3*c*((a+b*(d*x+c)^
(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(
1/3))+20*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3
))^2*sinh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3)
)-120*sinh(a+b*(d*x+c)^(1/3)))-8/b^6*a^7*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x
+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))-2*c^2*a*((a+b*(d*x+c)^(1/3))*cosh(a+b*(
d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))+70/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*co
sh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+12*(a
+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sinh(a+b
```

```
*(d*x+c)^(1/3))+24*cosh(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*((a+b*(d*x+c)^(1/3))
^3*cosh(a+b*(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+
6*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+b*(d*x+c)^(1/3)))-8/
b^6*a*((a+b*(d*x+c)^(1/3))^7*cosh(a+b*(d*x+c)^(1/3))-7*(a+b*(d*x+c)^(1/3))^
6*sinh(a+b*(d*x+c)^(1/3))+42*(a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-
210*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+840*(a+b*(d*x+c)^(1/3))^3
*cosh(a+b*(d*x+c)^(1/3))-2520*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))
+5040*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-5040*sinh(a+b*(d*x+c)^(1/
3)))+28/b^6*a^2*((a+b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+
c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))+30*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+
c)^(1/3))-120*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+360*(a+b*(d*x+c
)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-720*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)
^(1/3))+720*cosh(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*((a+b*(d*x+c)^(1/3))^2*cosh
(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+
b*(d*x+c)^(1/3)))+20/b^3*a^3*c*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3
))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3)))
-10/b^3*a^4*c*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)
^(1/3)))+10/b^3*c*a*((a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-4*(a+b*(
d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(
d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+24*cosh(a+b*(d
*x+c)^(1/3)))-20/b^3*c*a^2*((a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-3
*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cosh(a
+b*(d*x+c)^(1/3))-6*sinh(a+b*(d*x+c)^(1/3))))
```

Maxima [A] time = 1.18272, size = 867, normalized size = 1.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*sinh((d*x + c)^(1/3)*b + a) + (c^3*e^((d*x + c)^(1/3)*b + a)
/b - c^3*e^(-(d*x + c)^(1/3)*b - a)/b - 3*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c^2*e^((d*x + c)^(1/3)*b)/
b^4 + 3*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c
^2*e^(-(d*x + c)^(1/3)*b - a)/b^4 + 3*((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5
/3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x
 + c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*c*e^((d*x + c)^(
1/3)*b)/b^7 - 3*((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/
3)*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*
b + 720)*c*e^(-(d*x + c)^(1/3)*b - a)/b^7 - ((d*x + c)^3*b^9*e^a - 9*(d*x +
```

```
 c)^(8/3)*b^8*e^a + 72*(d*x + c)^(7/3)*b^7*e^a - 504*(d*x + c)^2*b^6*e^a +
3024*(d*x + c)^(5/3)*b^5*e^a - 15120*(d*x + c)^(4/3)*b^4*e^a + 60480*(d*x +
 c)*b^3*e^a - 181440*(d*x + c)^(2/3)*b^2*e^a + 362880*(d*x + c)^(1/3)*b*e^a
 - 362880*e^a)*e^((d*x + c)^(1/3)*b)/b^10 + ((d*x + c)^3*b^9 + 9*(d*x + c)^
(8/3)*b^8 + 72*(d*x + c)^(7/3)*b^7 + 504*(d*x + c)^2*b^6 + 3024*(d*x + c)^(
5/3)*b^5 + 15120*(d*x + c)^(4/3)*b^4 + 60480*(d*x + c)*b^3 + 181440*(d*x +
c)^(2/3)*b^2 + 362880*(d*x + c)^(1/3)*b + 362880)*e^(-(d*x + c)^(1/3)*b - a
)/b^10)*b)/d^3
```

Fricas [A] time = 2.0585, size = 467, normalized size = 0.87

$$\frac{3\left(\left(56\,b^6d^2x^2+72\,b^6cdx+18\,b^6c^2+\left(b^8d^2x^2+20160\,b^2\right)(dx+c)^{\frac{2}{3}}+240\left(7\,b^4dx+6\,b^4c\right)(dx+c)^{\frac{1}{3}}+40320\right)\cosh\left((dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 + (b^8*d^2*x^2 + 20160*b^2)*
(d*x + c)^(2/3) + 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) + 40320)*cosh((
d*x + c)^(1/3)*b + a) - 2*(3360*b^3*d*x + 3240*b^3*c + 12*(14*b^5*d*x + 9*b
^5*c)*(d*x + c)^(2/3) + (4*b^7*d^2*x^2 + 3*b^7*c*d*x + 20160*b)*(d*x + c)^(
1/3))*sinh((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sinh\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x**2*sinh(a + b*(c + d*x)**(1/3)), x)
```

Giac [B] time = 3.70743, size = 2919, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c
^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*
b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(
1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*
c - 2*((d*x + c)^(1/3)*b + a)*b^6*c^2 + 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b +
a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 5
6*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d
*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c
)^(1/3)*b + a)*a^7 + a^8 + 10*((d*x + c)^(1/3)*b + a)^4*b^3*c - 40*((d*x +
c)^(1/3)*b + a)^3*a*b^3*c + 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c - 40*((d
*x + c)^(1/3)*b + a)*a^3*b^3*c + 10*a^4*b^3*c + 2*b^6*c^2 - 8*((d*x + c)^(1
/3)*b + a)^7 + 56*((d*x + c)^(1/3)*b + a)^6*a - 168*((d*x + c)^(1/3)*b + a)
^5*a^2 + 280*((d*x + c)^(1/3)*b + a)^4*a^3 - 280*((d*x + c)^(1/3)*b + a)^3*
a^4 + 168*((d*x + c)^(1/3)*b + a)^2*a^5 - 56*((d*x + c)^(1/3)*b + a)*a^6 +
8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*
a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x +
 c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)
*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b
 + a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 + 120*((d*x + c)^(1/
3)*b + a)^2*b^3*c - 240*((d*x + c)^(1/3)*b + a)*a*b^3*c + 120*a^2*b^3*c - 3
36*((d*x + c)^(1/3)*b + a)^5 + 1680*((d*x + c)^(1/3)*b + a)^4*a - 3360*((d*
x + c)^(1/3)*b + a)^3*a^2 + 3360*((d*x + c)^(1/3)*b + a)^2*a^3 - 1680*((d*x
 + c)^(1/3)*b + a)*a^4 + 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*
a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a
 + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 +
 1680*a^4 + 240*b^3*c - 6720*((d*x + c)^(1/3)*b + a)^3 + 20160*((d*x + c)^(
1/3)*b + a)^2*a - 20160*((d*x + c)^(1/3)*b + a)*a^2 + 6720*a^3 + 20160*((d*
x + c)^(1/3)*b + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 - 40320
*(d*x + c)^(1/3)*b + 40320)*e^((d*x + c)^(1/3)*b + a)/(b^8*d^2) + (((d*x +
c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c
^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3
*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*
a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*c + 2*((d*x +
c)^(1/3)*b + a)*b^6*c^2 - 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x
 + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(
1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*
b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)
*a^7 + a^8 - 10*((d*x + c)^(1/3)*b + a)^4*b^3*c + 40*((d*x + c)^(1/3)*b + a
)^3*a*b^3*c - 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c + 40*((d*x + c)^(1/3)*
b + a)*a^3*b^3*c - 10*a^4*b^3*c + 2*b^6*c^2 + 8*((d*x + c)^(1/3)*b + a)^7 -
```

56*((d*x + c)^(1/3)*b + a)^6*a + 168*((d*x + c)^(1/3)*b + a)^5*a^2 - 280*((d*x + c)^(1/3)*b + a)^4*a^3 + 280*((d*x + c)^(1/3)*b + a)^3*a^4 - 168*((d*x + c)^(1/3)*b + a)^2*a^5 + 56*((d*x + c)^(1/3)*b + a)*a^6 - 8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x + c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b + a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 - 120*((d*x + c)^(1/3)*b + a)^2*b^3*c + 240*((d*x + c)^(1/3)*b + a)*a*b^3*c - 120*a^2*b^3*c + 336*((d*x + c)^(1/3)*b + a)^5 - 1680*((d*x + c)^(1/3)*b + a)^4*a + 3360*((d*x + c)^(1/3)*b + a)^3*a^2 - 3360*((d*x + c)^(1/3)*b + a)^2*a^3 + 1680*((d*x + c)^(1/3)*b + a)*a^4 - 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 240*b^3*c + 6720*((d*x + c)^(1/3)*b + a)^3 - 20160*((d*x + c)^(1/3)*b + a)^2*a + 20160*((d*x + c)^(1/3)*b + a)*a^2 - 6720*a^3 + 20160*((d*x + c)^(1/3)*b + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 + 40320*(d*x + c)^(1/3)*b + 40320)*e^(-(d*x + c)^(1/3)*b - a)/(b^8*d^2))/(b*d)

3.99 $\int x \sinh\left(a + b\sqrt[3]{c + dx}\right) dx$

Optimal. Leaf size=261

$$-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{360\sinh}{}$$

```
[Out] (-6*c*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*Cosh[a
+ b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)
^(1/3)])/(b*d^2) + (60*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (
3*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^2) - (360*Sinh[a + b*(c
 + d*x)^(1/3)])/(b^6*d^2) + (6*c*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)
])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^2)
- (15*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^2)
```

Rubi [A] time = 0.314847, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5364, 5286, 3296, 2638, 2637}

$$-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{6c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{360\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*(c + d*x)^(1/3)],x]
```


```
[Out] (-6*c*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*Cosh[a
+ b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)
^(1/3)])/(b*d^2) + (60*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (
3*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^2) - (360*Sinh[a + b*(c
 + d*x)^(1/3)])/(b^6*d^2) + (6*c*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)
])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^2)
- (15*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^2)
```


Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{\text{Subst}\left(\int(-c+x)\sinh\left(a+b\sqrt[3]{x}\right)dx,x,c+dx\right)}{d^2}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\left(-c+x^3\right)\sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}$$
$$= \frac{3\,\text{Subst}\left(\int\left(-cx^2\sinh(a+bx)+x^5\sinh(a+bx)\right)dx,x,\sqrt[3]{c+dx}\right)}{d^2}$$
$$= \frac{3\,\text{Subst}\left(\int x^5\sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2} - \frac{(3c)\,\text{Subst}\left(\int x^2\sinh(a+bx)\,dx,x,\sqrt[3]{c+}\right.}{d^2}$$
$$= -\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{15\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6c\sqrt[3]{c+dx}\,\text{si}}{}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60(c+dx)\cosh\left(a+b\right.}{b^3d^2}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60(c+dx)\cosh\left(a+b\right.}{b^3d^2}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+\right.}{bd^2}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+\right.}{bd^2}$$

Mathematica [A] time = 0.343548, size = 118, normalized size = 0.45

$$\frac{3b\left(b^4dx(c+dx)^{2/3}+2b^2(9c+10dx)+120\sqrt[3]{c+dx}\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)-3\left(b^4\sqrt[3]{c+dx}(3c+5dx)+60b^2(c+dx)^{2/3}+\right.}{b^6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3) + 2*b^2*(9*c + 10*d*x))
*Cosh[a + b*(c + d*x)^(1/3)] - 3*(120 + 60*b^2*(c + d*x)^(2/3) + b^4*(c + d
*x)^(1/3)*(3*c + 5*d*x))*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Maple [B] time = 0.007, size = 659, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^2/b^3*(1/b^3*((a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x
+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x
+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c
)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-120*sinh(a+b*(d*x+c)^(1/3)))-5/b^3*a*((a+b
*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*
(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*
x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+24*cosh(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*
((a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*sinh
(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+
b*(d*x+c)^(1/3)))-10/b^3*a^3*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))
-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3)))+5
/b^3*a^4*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3
)))-1/b^3*a^5*cosh(a+b*(d*x+c)^(1/3))-c*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*
x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c
)^(1/3)))+2*c*a*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+
c)^(1/3)))-c*a^2*cosh(a+b*(d*x+c)^(1/3)))
```

Maxima [A] time = 1.22952, size = 501, normalized size = 1.92

$$\frac{2\,d^2x^2\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(\frac{c^2e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}-\frac{c^2e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{2\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)ce^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}+\frac{2\left((dx+c)b^3\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*sinh((d*x + c)^(1/3)*b + a) - (c^2*e^((d*x + c)^(1/3)*b + a)
/b - c^2*e^(-(d*x + c)^(1/3)*b - a)/b - 2*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c*e^((d*x + c)^(1/3)*b)/b^
4 + 2*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c*e
^(-(d*x + c)^(1/3)*b - a)/b^4 + ((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5/3)*b^
5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x + c)^
(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*e^((d*x + c)^(1/3)*b)/
b^7 - ((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/3)*b^4 + 1
```

```
20*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*b + 720)*e
^(-(d*x + c)^(1/3)*b - a)/b^7)*b)/d^2
```

Fricas [A] time = 2.09024, size = 294, normalized size = 1.13

$$\frac{3\left(\left((dx+c)^{\frac{2}{3}}b^5dx+20\,b^3dx+18\,b^3c+120\,(dx+c)^{\frac{1}{3}}b\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(60\,(dx+c)^{\frac{2}{3}}b^2+\left(5\,b^4dx+3\,b^4c\right)(dx+\right.\right.}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(2/3)*b^5*d*x + 20*b^3*d*x + 18*b^3*c + 120*(d*x + c)^(1/3)*b
)*cosh((d*x + c)^(1/3)*b + a) - (60*(d*x + c)^(2/3)*b^2 + (5*b^4*d*x + 3*b^
4*c)*(d*x + c)^(1/3) + 120)*sinh((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x*sinh(a + b*(c + d*x)**(1/3)), x)
```

Giac [B] time = 2.25627, size = 953, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3/2*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c
+ a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 1
```

```
0*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*
x + c)^(1/3)*b + a)*a^4 + a^5 - 2*((d*x + c)^(1/3)*b + a)*b^3*c + 2*a*b^3*c
 + 5*((d*x + c)^(1/3)*b + a)^4 - 20*((d*x + c)^(1/3)*b + a)^3*a + 30*((d*x
+ c)^(1/3)*b + a)^2*a^2 - 20*((d*x + c)^(1/3)*b + a)*a^3 + 5*a^4 + 2*b^3*c
- 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2*a - 60*((d*x
+ c)^(1/3)*b + a)*a^2 + 20*a^3 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x +
 c)^(1/3)*b + a)*a + 60*a^2 - 120*(d*x + c)^(1/3)*b + 120)*e^((d*x + c)^(1/
3)*b + a)/(b^5*d) + (((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b
 + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*
b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^
2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 + 2*((d*x + c)^(1/3)*b + a)*b^3
*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3
*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*
a^4 + 2*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2
*a - 60*((d*x + c)^(1/3)*b + a)*a^2 + 20*a^3 - 60*((d*x + c)^(1/3)*b + a)^2
 + 120*((d*x + c)^(1/3)*b + a)*a - 60*a^2 - 120*(d*x + c)^(1/3)*b - 120)*e^
(-(d*x + c)^(1/3)*b - a)/(b^5*d))/(b*d)
```

3.100 $\int \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=85

$$-\frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[Out] (6*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Cosh[a + b*(c
+ d*x)^(1/3)])/(b*d) - (6*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2
*d)
```

Rubi [A] time = 0.0790904, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5310, 5304, 3296, 2638}

$$-\frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Cosh[a + b*(c
+ d*x)^(1/3)])/(b*d) - (6*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2
*d)
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rule 5304

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Sinh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{\text{Subst}\left(\int \sinh\left(a+b\sqrt[3]{x}\right)dx, x, c+dx\right)}{d}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\,\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd}$$
$$= \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{6\,\text{Subst}\left(\int \sinh(a+\right.}{b^2d}$$
$$= \frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right.}{b^2d}$$

Mathematica [A] time = 0.0819836, size = 65, normalized size = 0.76

$$\frac{3\left(b^2(c+dx)^{2/3}+2\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)-6b\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(2 + b^2*(c + d*x)^(2/3))*Cosh[a + b*(c + d*x)^(1/3)] - 6*b*(c + d*x)^(1
/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A] time = 0.007, size = 133, normalized size = 1.6

$$3\,\frac{\left(a+b\sqrt[3]{dx+c}\right)^2\cosh\left(a+b\sqrt[3]{dx+c}\right)-2\,\left(a+b\sqrt[3]{dx+c}\right)\sinh\left(a+b\sqrt[3]{dx+c}\right)+2\,\cosh\left(a+b\sqrt[3]{dx+c}\right)-2\,a\left(\left(a+b\sqrt[3]{\right.\right.}{db^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d/b^3*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3)
)*sinh(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3))-2*a*((a+b*(d*x+c)^(1/3)
)*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))+a^2*cosh(a+b*(d*x+c)^(1/
3)))
```

Maxima [A] time = 1.15613, size = 185, normalized size = 2.18

$$-\frac{b\left(\frac{\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}-\frac{\left((dx+c)b^3+3\,(dx+c)^{\frac{2}{3}}b^2+6\,(dx+c)^{\frac{1}{3}}b+6\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^4}\right)-2\,(dx+c)\sinh\left((dx+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*((((d*x + c)*b^3*e^a - 3*(d*x + c)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)
*b*e^a - 6*e^a)*e^((d*x + c)^(1/3)*b)/b^4 - ((d*x + c)*b^3 + 3*(d*x + c)^(2
/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*e^(-(d*x + c)^(1/3)*b - a)/b^4) - 2*(d*x
 + c)*sinh((d*x + c)^(1/3)*b + a))/d
```

Fricas [A] time = 2.05477, size = 159, normalized size = 1.87

$$-\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2+2\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*(2*(d*x + c)^(1/3)*b*sinh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2
+ 2)*cosh((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] time = 1.73817, size = 94, normalized size = 1.11

$$\begin{cases} x\sinh(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x\sinh\left(a + b\sqrt[3]{c}\right) & \text{for } d = 0 \\ \frac{3(c+dx)^{\frac{2}{3}}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sinh(a + b*c**(
1/3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*cosh(a + b*(c + d*x)**(1/3))/(b*d) -
6*(c + d*x)**(1/3)*sinh(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cosh(a + b*(c
+ d*x)**(1/3))/(b**3*d), True))
```

Giac [A] time = 1.68657, size = 173, normalized size = 2.04

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 - 2\left((dx+c)^{\frac{1}{3}}b+a\right)a + a^2 - 2\,(dx+c)^{\frac{1}{3}}b + 2\right)e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{2\,b^3d} + \frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 - 2\left((dx+c)^{\frac{1}{3}}b+\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2*(d*x
 + c)^(1/3)*b + 2)*e^((d*x + c)^(1/3)*b + a)/(b^3*d) + 3/2*(((d*x + c)^(1/3
)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 + 2*(d*x + c)^(1/3)*b + 2)*e
^(-(d*x + c)^(1/3)*b - a)/(b^3*d)
```

3.101 $\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=232

$$\sinh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)+\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Ch}$$

```
[Out] CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))]*Sinh[a + b*c^(1/3)] + CoshInteg
ral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]*Sinh[a - (-1)^(1/3)*b*c^(1/3)
] + CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))]*Sinh[a + (-1)
^(2/3)*b*c^(1/3)] - Cosh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)
^(1/3))] - Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3
) - (c + d*x)^(1/3))] + Cosh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)
^(1/3)*c^(1/3) + (c + d*x)^(1/3))]
```

Rubi [A] time = 0.518501, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5364, 5292, 3303, 3298, 3301}

$$\sinh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)+\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Ch}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))]*Sinh[a + b*c^(1/3)] + CoshInteg
ral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]*Sinh[a - (-1)^(1/3)*b*c^(1/3)
] + CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))]*Sinh[a + (-1)
^(2/3)*b*c^(1/3)] - Cosh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)
^(1/3))] - Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3
) - (c + d*x)^(1/3))] + Cosh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)
^(1/3)*c^(1/3) + (c + d*x)^(1/3))]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx &= \text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt[3]{x}\right)}{-c+x}\,dx, x, c+dx\right)\\
&= 3\,\text{Subst}\left(\int \frac{x^2\sinh(a+bx)}{-c+x^3}\,dx, x, \sqrt[3]{c+dx}\right)\\
&= 3\,\text{Subst}\left(\int \left(-\frac{\sinh(a+bx)}{3\left(\sqrt[3]{c}-x\right)}-\frac{\sinh(a+bx)}{3\left(-\sqrt[3]{-1}\sqrt[3]{c}-x\right)}-\frac{\sinh(a+bx)}{3\left((-1)^{2/3}\sqrt[3]{c}-x\right)}\right)dx, x, \sqrt[3]{c+dx}\right)\\
&= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)-\text{Subst}\left(\int \frac{\sinh(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)-\text{Sub}\\
&= \cosh\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)+\left(i\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\right)\text{Subs}\\
&= \textbf{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sinh\left(a+b\sqrt[3]{c}\right)+\textbf{Chi}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)+
\end{aligned}$$

Mathematica [C] time = 0.0679446, size = 233, normalized size = 1.

$$\frac{1}{2}\left(\text{RootSum}\left[c-\#1^3\&,\sinh(\#1b+a)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\cosh(\#1b+a)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(\#1b+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] (-RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)
] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] - Cosh[a + b*#1]*
SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((c
+ d*x)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[
b*((c + d*x)^(1/3) - #1)] + CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a +
 b*#1] + Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b
*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sinh\left(a+b\sqrt[3]{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/3))/x,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/3))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x, x)
```

Fricas [B] time = 2.42165, size = 1539, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] -1/2*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^
3*c)^(1/3)*(sqrt(-3) + 1) - a) + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1
/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - 1/2*Ei(-(
d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)
*(sqrt(-3) - 1) + a) + 1/2*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(
-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 1/2*Ei(-(d*x + c)^(
1/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) + 1/2*Ei((d*x + c)^(1/3)*b
+ (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) - 1/2*Ei(-(d*x + c)^(1/3)*b - 1
/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a)
 + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(
-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + 1/2*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)
^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - 1/2*Ei(
(d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-b^3*c)^(1
/3)*(sqrt(-3) - 1) - a) + 1/2*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*sinh(a
 + (b^3*c)^(1/3)) - 1/2*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-
b^3*c)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + b\sqrt[3]{c + dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3))/x,x)
```

```
[Out] Integral(sinh(a + b*(c + d*x)**(1/3))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
```

```
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x, x)
```

3.102 $\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=329

$$\frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}+\frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{\sqrt[3]{-1}bd}{}$$

```
[Out] (b*d*Cosh[a + b*c^(1/3)]*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^
(2/3)) + ((-1)^(2/3)*b*d*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-(b*((
-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Cosh[
a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1
/3))])/(3*c^(2/3)) - Sinh[a + b*(c + d*x)^(1/3)]/x - (b*d*Sinh[a + b*c^(1/3
)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(2/3)*b
*d*Sinh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (c +
 d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Sinh[a - (-1)^(1/3)*b*c^(1/3)]
*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))])/(3*c^(2/3))
```

Rubi [A] time = 0.724394, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5364, 5288, 5281, 3303, 3298, 3301}

$$\frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}+\frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{\sqrt[3]{-1}bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (b*d*Cosh[a + b*c^(1/3)]*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^
(2/3)) + ((-1)^(2/3)*b*d*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-(b*((
-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Cosh[
a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1
/3))])/(3*c^(2/3)) - Sinh[a + b*(c + d*x)^(1/3)]/x - (b*d*Sinh[a + b*c^(1/3
)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(2/3)*b
*d*Sinh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (c +
 d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Sinh[a - (-1)^(1/3)*b*c^(1/3)]
*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))])/(3*c^(2/3))
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx = d\,\text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt[3]{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (3d)\,\text{Subst}\left(\int \frac{x^2\sinh(a+bx)}{\left(c-x^3\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{c-x^3}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\,\text{Subst}\left(\int\left(\frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-x\right)} + \frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}+\sqrt[3]{-1}x\right)} + \frac{\cosh(}{3c^{2/3}\left(\sqrt[3]{c}\right.}\right.\right.$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cosh(a+bx}{\sqrt[3]{c}+\sqrt[3]{-1}x}\right.}{3c^{2/3}}$$

$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(bd}{}$$

$$= \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\sqrt[3]{-1}bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Chi}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\right.}{3c^{2/3}}$$

Mathematica [C] time = 1.83551, size = 210, normalized size = 0.64

$$\frac{e^{-a}\left(bdx\text{RootSum}\left[c-\#1^3\&, \frac{-\sinh(\#1b)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\cosh(\#1b)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(\#1b)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)-\cosh(\#1b)\text{Shi}\left(b\left(\sqrt[3]{c+\right.\right.}{\#1^2}\right.\right.}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (b*d*x*RootSum[c - #1^3 & , (E^(a + b*#1)*ExpIntegralEi[b*((c + d*x)^(1/3)
- #1)])/#1^2 & ] + (3/E^(b*(c + d*x)^(1/3)) - 3*E^(2*a + b*(c + d*x)^(1/3))
 + b*d*x*RootSum[c - #1^3 & , (Cosh[b*#1]*CoshIntegral[b*((c + d*x)^(1/3) -
 #1)] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[b*#1] - Cosh[b*#1]*Sinh
Integral[b*((c + d*x)^(1/3) - #1)] + Sinh[b*#1]*SinhIntegral[b*((c + d*x)^(
1/3) - #1)])/#1^2 & ])/E^a)/(6*x)
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sinh\left(a + b\sqrt[3]{dx + c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/3))/x^2,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/3))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50018, size = 2032, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a + (
b^3*c)^(1/3)) - 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))
*cosh(-a + (-b^3*c)^(1/3)) - 2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b
^3*c)^(1/3))*sinh(a + (b^3*c)^(1/3)) + 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1
/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b^3*c)^(1/3)) - (b^3*c)^(1/3)*(sqrt(-3)
*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(
1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)
*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c
)^(1/3)*(sqrt(-3) + 1) + a) + (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x +
 c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)*(sqr
```

```
t(-3) - 1) + a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b
+ 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1)
 - a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3
*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) + (-b^
3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(
sqrt(-3) + 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - (b^3*c)^(1/3)*
(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) + (-b^3*c)^(1/3)*(sqrt(-3)*d
*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/
2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 12*c*sinh((d*x + c)^(1/3)*b + a))/(c
*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a + b\sqrt[3]{c + dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3))/x**2,x)
```

```
[Out] Integral(sinh(a + b*(c + d*x)**(1/3))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left((dx + c)^{\frac{1}{3}}b + a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                 #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```